

Optimum Load of WPT System Analyzed by S-Parameters

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Abstract—In this paper, the transfer efficiency of WPT system is calculated by using S-parameters if a WPT system is equivalent to a 2-ports lossy network. The optimum source and load impedances are deduced from the conjugated matching conditions for the maximum transfer efficiency of WPT system. The approach is applied to analyze the effects of different transmitting/receiving elements, different load on the transfer efficiency of WPT system. It is found that the method is very general and practical to deal with different WPT systems because the S-parameter can be obtained easily by the measurement or by the numerical simulation.

I. INTRODUCTION

Wireless power transfer (WPT) is interested again because of its potential application to charge laptops, cell-phones, household robots, MP3 players and other portable electronics without cords [1]-[6]. The optimum load for maximum transfer efficiency of WPT system was presented when the WPT system was equivalent to a 2-ports lossy network in [5] by present authors. The maximum transfer efficiency of WPT system with resonant transmitting/receiving elements and non-resonant transmitting/receiving elements have been analyzed and compared in [1]. It has been observed that the resonant characteristic of transmitting/receiving elements gives little impact on the maximum transfer efficiency when the receiving element is placed close to the transfer element, but affects the maximum transfer efficiency greatly when the receiving element is placed far from the transmitting element.

II. THREE WPT SYSTEMS

Three WPT systems used in this report are shown in Fig. 1. In type-A, a square loop D with a side length of 30 cm is used as the transmitting element and receiving element; In type-B, the same square loop D as type-A loaded with a 210 nF inductor is used as the transmitting element and receiving element; In type-C, the same square loop D with a parasitic square helical coil C is used as the transmitting element and receiving element; All transmitting or receiving elements are made of copper wire with the conductivity of $\sigma = 5.8 \times 10^7$ [S/m]. The radius of all wires are 2 mm. The input impedances of three types of elements which are used in the WPT systems are compared in Fig. 2. It is found that the type-A element does not resonate at the concerned frequency range, while the type-B element gets antiresonance and that the type-C element gets resonance at a frequency of 19.2MHz, respectively. Frequency

of 19.2MHz will be regarded as the operating frequency in this report.

III. MAXIMUM POWER TRANSFER EFFICIENCY OF WPT SYSTEM

There are several power gain definitions for a 2-ports network which is shown in Fig. 3 [7]. The operating power gain G_p , the transducer power gain G_t and the available power gain G_a will be introduced in this section .

A. Available Power Gain G_a

The available power gain is the ratio of the incident power P_{2b} to the load to the incident power P_{1a} at port 1, is defined as

$$G_a = \frac{P_{2b}}{P_{1a}} = \frac{|b_2|^2}{|a_1|^2} = \frac{|s_{21}|^2}{|1 - s_{22}\Gamma_l|^2} \quad (1)$$

where, Γ_l is the reflection coefficient related with the load impedance Z_l and calculated by

$$\Gamma_l = \frac{Z_l - Z_0}{Z_l + Z_0}, \quad (2)$$

B. Transducer Power Gain G_t

The transducer power gain is the ratio of power P_2 delivered to the load to the incident power P_{1a} at port 1, is defined as

$$\begin{aligned} G_t &= \frac{P_2}{P_{1a}} = \frac{|b_2|^2 - |a_2|^2}{|a_1|^2} \\ &= \frac{|s_{21}|^2(1 - |\Gamma_l|^2)}{|1 - s_{22}\Gamma_l|^2} \\ &= G_a(1 - |\Gamma_l|^2) \end{aligned} \quad (3)$$

C. Operating Power Gain G_p

The operating power gain is the ratio of the power P_2 delivered to the load Z_l to the input power P_1 at the input port(port 1), is defined as

$$\begin{aligned} G_p &= \frac{P_2}{P_1} = \frac{|b_2|^2 - |a_2|^2}{|a_1|^2 - |b_1|^2} \\ &= \frac{|b_2|^2 - |a_2|^2}{|a_1|^2(1 - |\Gamma_{in}|^2)} \\ &= \frac{G_t}{1 - |\Gamma_{in}|^2} \\ &= \frac{(1 - |\Gamma_l|^2)|s_{21}|^2}{|1 - s_{22}\Gamma_l|^2 - |s_{11} - \Delta\Gamma_l|^2} \end{aligned} \quad (4)$$

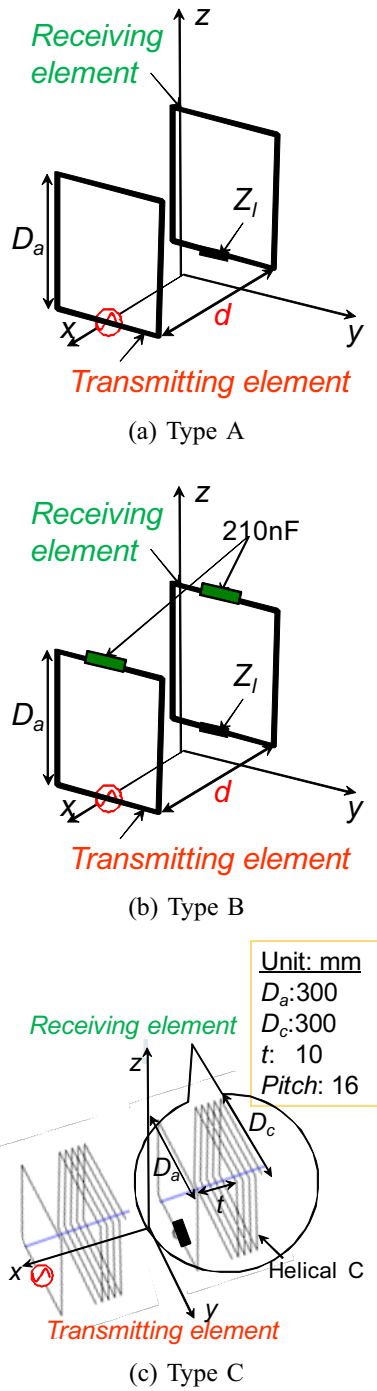


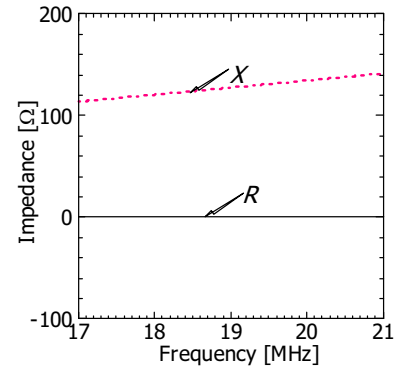
Fig. 1. Three types of WPT systems.

where Γ_{in} is the reflection coefficient at the input port, calculated by

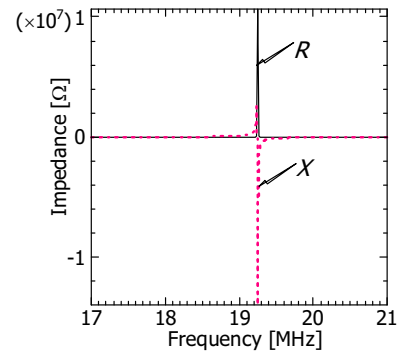
$$\Gamma_{in} = s_{11} + \frac{s_{12}s_{21}\Gamma_l}{1 - s_{22}\Gamma_l} = \frac{s_{11} - \Delta\Gamma_l}{1 - s_{22}\Gamma_l}, \quad (5)$$

and

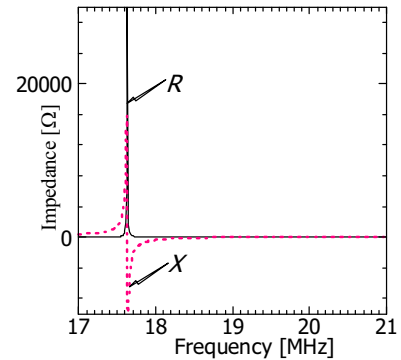
$$\Delta = s_{11}s_{22} - s_{12}s_{21}. \quad (6)$$



(a) Type A



(b) Type B



(c) Type C

Fig. 2. Input impedances of three types of transmitting elements.

D. Maximum Transfer Power Efficiency and Matching Condition

The gains described in equations (1), (3) and (4) is nothing else but the power transfer efficiency from input port to output port(port 1). Therefore, there are different efficiencies according to different gains. In this report, η_a , η_t and η_p will be introduced as defined as the following,

$$\eta_a = G_a, \quad (7)$$

$$\eta_t = G_t, \quad (8)$$

$$\eta_p = G_p, \quad (9)$$

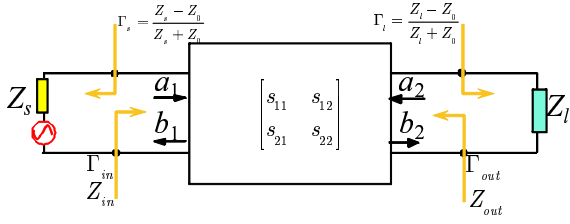


Fig. 3. 2-ports network.

and correspondingly called η_a as available power transfer efficiency, η_t as transducer power transfer efficiency and η_p as operating power transfer efficiency, respectively. In general, $\eta_p \geq \eta_t$, $\eta_a \geq \eta_t$. However, $\eta_a = \eta_t = \eta_p = \eta_{max}$ when both the input and output ports are conjugately matched as

$$\Gamma_s = \Gamma_{in}^*, \quad (10)$$

$$\Gamma_l = \Gamma_{out}^*, \quad (11)$$

where, Γ_s is the reflection coefficient related with the source impedance Z_s and calculated by

$$\Gamma_s = \frac{Z_s - Z_0}{Z_s + Z_0}. \quad (12)$$

and Γ_{out} is the reflection coefficient at the output port, calculated by

$$\Gamma_{out} = s_{22} + \frac{s_{12}s_{21}\Gamma_s}{1 - s_{11}\Gamma_s} = \frac{s_{22} - \Delta\Gamma_s}{1 - s_{11}\Gamma_s} \quad (13)$$

From the matching conditions (10) and (11), the following equations for optimum Γ_{sop} and Γ_{lop} can be deduced.

$$C_1\Gamma_{sop}^2 + B_1\Gamma_{sop} + C_1^* = 0, \quad (14)$$

$$C_2\Gamma_{lop}^2 + B_2\Gamma_{lop} + C_2^* = 0 \quad (15)$$

where,

$$B_1 = 1 + |s_{11}|^2 - |s_{22}|^2 - |\Delta|^2, \quad (16)$$

$$C_1 = s_{11} - |\Delta|s_{22}^*, \quad (17)$$

$$B_2 = 1 + |s_{22}|^2 - |s_{11}|^2 - |\Delta|^2, \quad (18)$$

$$C_2 = s_{22} - |\Delta|s_{11}^*. \quad (19)$$

Therefore, the optimum Γ_{sop} and Γ_{lop} can be obtained by

$$\Gamma_{sop} = \frac{-B_1 \pm \sqrt{B_1^2 - 4|C_1|^2}}{2C_1}, \quad (20)$$

$$\Gamma_{lop} = \frac{-B_2 \pm \sqrt{B_2^2 - 4|C_2|^2}}{2C_2}. \quad (21)$$

The optimum load impedance Z_{lop} and source impedance Z_{sop} when the transfer efficiency achieves the maximum can be obtained easily from equation (2) and (12). Both the optimum load impedance and source impedance only depend on the S-parameters of the 2-ports network, in other word, only be determined by the 2-port network structure.

There are two solutions for both Γ_{sop} and Γ_{lop} in (20) and (21), however, the solution should be selected the one which satisfies $|\Gamma_{sop}| \leq 1$ or $|\Gamma_{lop}| \leq 1$.

IV. POWER TRANSFER EFFICIENCY OF WPT SYSTEM CALCULATED BY USING S-PARAMETERS

A. η_{max} with Conjugated Matching Condition

The maximum transfer efficiencies of three WPT systems when the load impedance and source impedance are satisfied with their matching condition respectively are shown in Figs. 4-7, where all S-parameters are calculated by FEKO soft. From Figs. 4-7, the following results can be observed.

(1) The maximum transfer efficiencies of three WPT systems are greater than 95% when the distance d between transmitting element and receiving element is equivalent to 10 cm no matter how the resonance situation of the transmitting/receiving element of the WPT system is.

(2) The maximum transfer efficiency decreases when the distance d is increased. The reduction in transfer efficiency of type B is the smallest, followed by type C and type A.

(3) The maximum transfer efficiency of type C is the most sensitive to the frequency, indicating that type C has narrow band for high transfer efficiency.

Therefore, it can be said that the maximum transfer efficiency depends on the transmitting and receiving element structure even if they are matched conjugately both at the input and output ports. From the above results, type B maybe the best selection, however it should be noted that in type B, a lumped inductance of 210nF is used without considering its loss. The more discussions on effect of the loss of lumped impedance on the transfer efficiency can be found in [8].

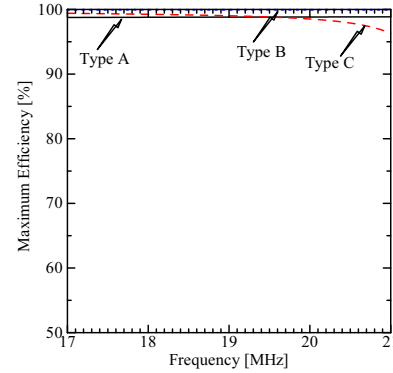


Fig. 4. η_{max} of three WPT systems ($d=0.1m$).

B. η_t with Conjugated Matching Condition only at Output Port

In practical case, the source impedance Z_s is usually to be 50Ω , the conjugated matching condition at the output port of

$$\Gamma_l = \Gamma_{out}^*, \quad (22)$$

will be

$$\Gamma_l = s_{22}^*. \quad (23)$$

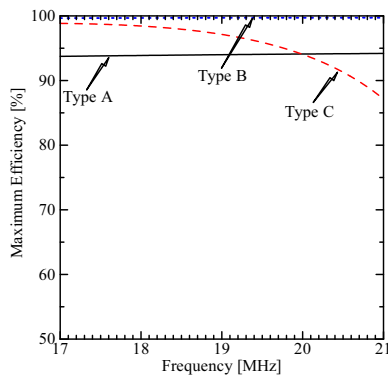


Fig. 5. η_{max} of three WPT systems ($d=0.3m$).

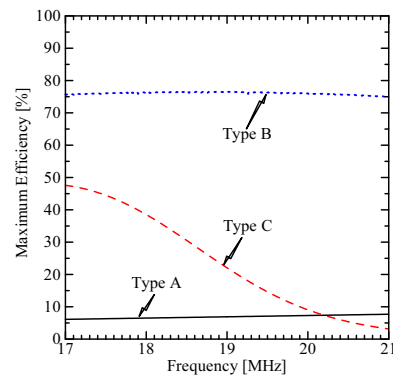


Fig. 7. η_{max} of three WPT systems ($d=1.5m$).

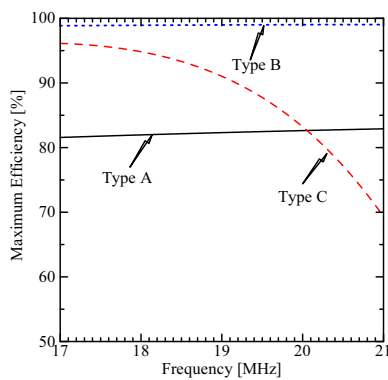


Fig. 6. η_{max} of three WPT systems ($d=0.5m$).

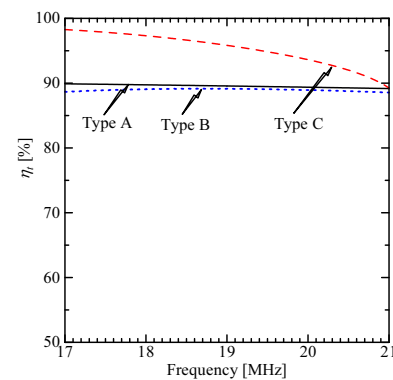


Fig. 8. η_t of three WPT systems ($d=0.1m$).

The transducer power transfer efficiency with the matching condition of (23) is very easily calculated by known S-parameters by using (12). The calculated transducer power transfer efficiencies of three types of WPT systems are shown in Figs. 8-11.

Comparing with η_{max} which are shown in Figs.4-7, the following conclusions can be observed,

- (1) $\eta_t \leq \eta_{max}$ for all cases.
- (2) η_t of type C is the highest one among three types WPT systems for any cases with different distance between the transmitting element and receiving element.

Therefore, from the calculated η_{max} and η_t in Figs. 4-11, it can be concluded that the transfer power efficiency depends not only on the matching conditions, but also on the structures of transmitting element and receiving element. To achieve the highest power transfer efficiency, there must be an optimum structure with optimum load impedance and source impedance for WPT systems.

V. CONCLUSIONS

In this report, the transfer efficiency of WPT system has been calculated by using S-parameters when a WPT system is equivalent to a 2-ports lossy network. Three different definitions of transfer efficiencies has been introduced, and the maximum transfer efficiency of WPT system only achieves when

the input port and output port are both matched conjugately. The approach described in this report has been applied to analyze the effects of different transmitting/receiving elements, different load on the transfer efficiency of WPT system. It has been found that the method is very general and practical to deal with different WPT systems because the S-parameter can be obtained easily by the measurement or by the numerical simulation. Moreover, it has been concluded that the transfer power efficiency depend not only on the matching conditions, but also on the structures of transfer element and receiving element.

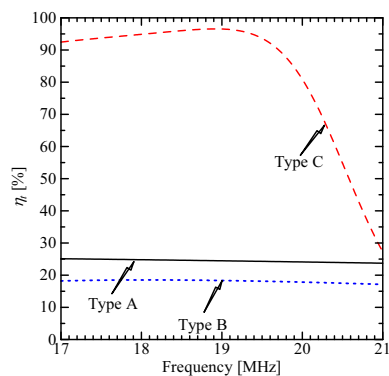


Fig. 9. η_t of three WPT systems ($d=0.3m$).

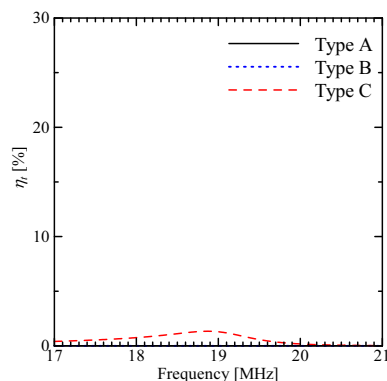


Fig. 11. η_t of three WPT systems ($d=1.5m$).

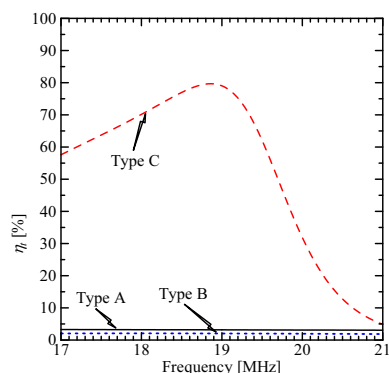


Fig. 10. η_t of three WPT systems ($d=0.5m$).

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