

# Analysis of Near-Field Power Transfer Using Scattering Parameters

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**Abstract**—Numerical analysis of the near-field power transfer for multi-user is performed by using the multi-port scattering parameters. The transfer power efficiency is derived by using the multi-port scattering parameters. Since the multi-port scattering parameters are easily measured by a vector network analyzer and calculated by a full-wave numerical analysis, it is convenient to investigate the relation between the power transfer efficiency and the geometry of the power transfer system such as relative positions between transmitting antennas and receiving antennas, antenna geometry, impedance matching of the antennas, and ohmic loss in antennas, which are important parameters to develop a near-field power transfer system with high power transfer efficiency.

## I. INTRODUCTION

Wireless power transfer (WPT) attracts great attention because of its potential application to charge laptop computers, cell-phones, household robots, portable music players and other portable electronic devices without cords. It was experimentally demonstrated that very efficient power transmission can be achieved by using the so-called evanescent resonant-coupling method, showing its potential for practical application [1]. It was shown that the evanescent resonant-coupling method can transmit energy for a longer distance than the previously used near-field induction method [2]. The evanescent resonant-coupling method was shown to be more efficient than the far-field radiation method, wherein the vast majority of the energy was wasted, due to the transmission loss [3]-[5].

The power transfer efficiency (PTE) is one of the most important parameters to evaluate the performance of the WPT system. In order to develop a WPT system with a high PTE, it is required to have an efficient method to calculate the PTE by analyzing electromagnetically the WPT system. If the transmitting antenna and receiving antenna are described as a two-port network circuit, the power transfer between the transmitting antenna and receiving antenna in the WPT system can be indicated by using the scattering parameters of the network circuit and further the PTE can be calculated by using the scattering parameters. The scattering parameters of a WPT system can be measured by a vector network analyzer and calculated by a full-wave numerical analysis. Therefore, the scattering parameters are a very efficient tool in analyzing and designing the antennas and RF modules for a WPT system.

A fundamental study focused on the PTE of a WPT system composed of dipole and loop antennas as the transmitting and

receiving antennas was carried out by the present authors, where a two-port scattering parameters calculated by the method of moments were used to analyze the system and it was found the largest PTE was obtained when the near-field coupled antennas of both transmitting and receiving sides were conjugate-matched with the impedance of the transmitting and receiving circuits, respectively [6]. The optimum load for maximum transfer efficiency of a practical WPT system was derived when the WPT system was equivalent to a 2-ports lossy network [7] also by the present authors.

In this research, multi-port scattering parameters are applied to the analysis of antennas in a WPT system corresponding to multi-user situations. The expression of PTE is defined in term of the multi-port scattering parameters. Some numerical simulations are shown to demonstrate it is easy to evaluate the PTE for various models of the antenna geometries, locations of transmitting and receiving antennas in a multi-user WPT system by using the multi-port scattering parameters.

## II. ANALYSIS OF MULTI-PORT NETWORK FOR MULTI-USER WPT

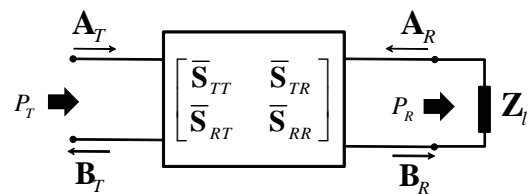


Fig. 1. Scattering parameters of a generalized 2-port network for multi-port network.

A general two-port network circuit is shown in Fig.1, where scattering parameters of the general two-port network circuit are defined. If the two-port network circuit is further divided into a group of transmitters with  $N_T$  ports at input port and a group of receivers with  $N_R$  ports at output port, this circuit is capable of expressing a multi-port network with  $N_T$  input ports and  $N_R$  output ports. Therefore, the relationship of the incident wave  $\mathbf{A}_T$ , reflected wave  $\mathbf{B}_T$  at the transmitter group, the incident wave  $\mathbf{A}_R$  and the reflected wave  $\mathbf{B}_R$  at the receiver group can be expressed by using the scattering parameters as

$$\begin{bmatrix} \mathbf{B}_T \\ \mathbf{B}_R \end{bmatrix} = \begin{bmatrix} \mathbf{S}_{TT} & \mathbf{S}_{TR} \\ \mathbf{S}_{RT} & \mathbf{S}_{RR} \end{bmatrix} \begin{bmatrix} \mathbf{A}_T \\ \mathbf{A}_R \end{bmatrix} \quad (1)$$

where,  $\mathbf{A}_T$  and  $\mathbf{B}_T$  are  $N_T$  vectors,  $\mathbf{A}_R$  and  $\mathbf{B}_R$  are  $N_R$  vectors.  $\mathbf{S}_{TT}$ ,  $\mathbf{S}_{TR}$ ,  $\mathbf{S}_{RT}$ ,  $\mathbf{S}_{RR}$  are  $N_T \times N_T$ ,  $N_T \times N_R$ ,  $N_R \times N_T$ ,  $N_R \times N_R$  matrix, respectively.

The incident wave  $\mathbf{A}_R$  at the receiving ports can be expressed by the reflecting matrix  $\Gamma_l$  and the reflected wave of  $\mathbf{B}_R$  as follows

$$\mathbf{A}_R = \Gamma_l \mathbf{B}_R \quad (2)$$

When the receiving ports are connected with the load of  $z_1, z_2, \dots, z_{N_r}$ , the matrix  $\Gamma_l$  becomes a  $N_R \times N_R$  diagonal matrix as follows.

$$\Gamma_l = \begin{bmatrix} \frac{z_1 - z_0}{z_1 + z_0} & 0 & \dots & 0 \\ 0 & \frac{z_2 - z_0}{z_2 + z_0} & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 \dots & \frac{z_{N_R} - z_0}{z_{N_R} + z_0} \end{bmatrix} \quad (3)$$

where the value in the diagonal element represents the reflection coefficient of each receiving port, respectively.

The following equation will be obtained if the equation (2) is substituted into equation (1).

$$\mathbf{B}_R = \{\Gamma_l - \mathbf{S}_{RR}\Gamma_l\}^{-1} \mathbf{S}_{RT}\mathbf{A}_T = \mathbf{D}\mathbf{A}_T \quad (4)$$

where,

$$\mathbf{D} = \{\mathbf{I} - \mathbf{S}_{RR}\Gamma_l\}^{-1} \mathbf{S}_{RT}. \quad (5)$$

Furthermore, if the equation (4) is substituted into equation (2), the vector of incident wave at receiving group can be expressed by the incident wave at transmitting group as

$$\mathbf{A}_R = \Gamma_l \{\mathbf{I} - \mathbf{S}_{RR}\Gamma_l\}^{-1} \mathbf{S}_{RT}\mathbf{A}_T = \Gamma_l \mathbf{D}\mathbf{A}_T \quad (6)$$

By substituting equation (6) into equation (1),  $\mathbf{B}_T$  can be expressed by

$$\mathbf{B}_T = \{\mathbf{S}_{TT} + \mathbf{S}_{TR}\Gamma_l \mathbf{D}\} \mathbf{A}_T = \Gamma_T \mathbf{A}_T \quad (7)$$

where,

$$\Gamma_T = \mathbf{S}_{TT} + \mathbf{S}_{TR}\Gamma_l \mathbf{D} \quad (8)$$

The total input power at the transmitting ports is the difference of incident wave with the reflection wave and is shown as

$$P_T = \frac{1}{2} \{\mathbf{A}_T^H \mathbf{A}_T - \mathbf{B}_T^H \mathbf{B}_T\} = \frac{1}{2} \mathbf{A}_T^H \{\mathbf{I} - \Gamma_T^H \Gamma_T\} \mathbf{A}_T. \quad (9)$$

and, the total power of the receiving ports is

$$P_R = \frac{1}{2} \{\mathbf{B}_R^H \mathbf{B}_R - \mathbf{A}_R^H \mathbf{A}_R\} = \frac{1}{2} \mathbf{A}_T^H \mathbf{D}^H \{\mathbf{I} - \Gamma_l^H \Gamma_l\} \mathbf{D} \mathbf{A}_T. \quad (10)$$

Therefore, the power transfer efficiency between total transmitting power and receiving power can be calculated by

$$\eta_p = \frac{P_R}{P_T} = \frac{\mathbf{A}_T^H \mathbf{D}^H \{\mathbf{I} - \Gamma_l^H \Gamma_l\} \mathbf{D} \mathbf{A}_T}{\mathbf{A}_T^H \{\mathbf{I} - \Gamma_T^H \Gamma_T\} \mathbf{A}_T}. \quad (11)$$

which is used in the following numerical analysis.

### III. POWER TRANSFER EFFICIENCY OF MULTI-USER WPT

A WPT system composed of one transmitting antenna and three receiving antennas is shown in Fig. 2 as a analysis model for following numerical simulation. The transmitting and receiving antennas have the same structure, which consists of a square wire loop with a side length of  $l$  and a  $N$ -turn square helix as parasitic element with pitch of  $s$ , as shown in Fig. 3. The distance between loop antenna and parasitic helix is  $d$ . In this study,  $N=4.5$ ,  $s=2$  cm, and  $d=2.5$  cm. The wire antennas are made of copper with the conductivity of  $\sigma = 5.8 \times 10^7$  S/m and radius of  $a = 2$  mm. The location of the transmitting and receiving antennas are defined in  $xyz$  coordinates and the center locations of the antennas are given in table I.

TABLE I  
LOCATIONS OF ANTENNA CENTERS

| Case 1       | Transmitter | Receiver A | Receiver B   | Receiver C |
|--------------|-------------|------------|--------------|------------|
| (x,y,z) [cm] | (0,0,0)     | (0,0,25)   | (0,25,-30)   | (75,0,10)  |
| Case 2       | Transmitter | Receiver A | Receiver B   | Receiver C |
| (x,y,z) [cm] | (0,0,0)     | (0,0,50)   | (0,50,-60)   | (150,0,20) |
| Case 3       | Transmitter | Receiver A | Receiver B   | Receiver C |
| (x,y,z) [cm] | (0,0,0)     | (0,0,100)  | (0,100,-120) | (300,0,40) |

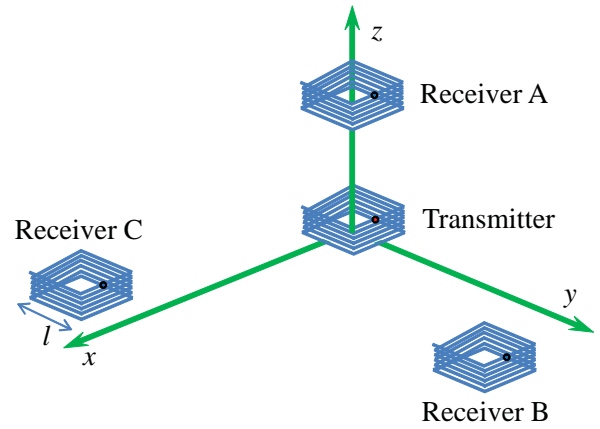


Fig. 2. Analysis model: WPT with one transmitting and three receiving antennas

The transfer efficiency of the WPT system is analyzed by using a 4-port circuit as shown in Fig. 4 and the equations shown in the above section. The scattering parameters of the network circuit were calculated by using the method of moments. The PTE of the WPT is shown in Fig. 5 for different positions of antennas. It is found the value of PTE and its bandwidth is largely dependent on the antenna positions between transmitting and receiving antennas. Fig. 6 shows the PTE of Case 2 with different antenna size. It is found that the frequency for the maximum PTE is dependent on antenna size,

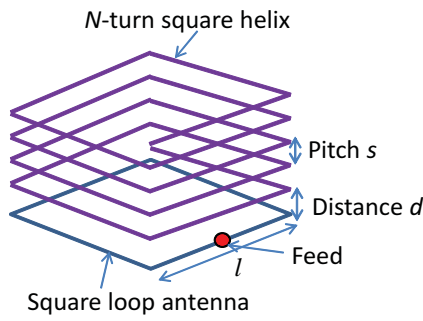


Fig. 3. Antenna geometry for transmitting and receiving antennas in WPT system.

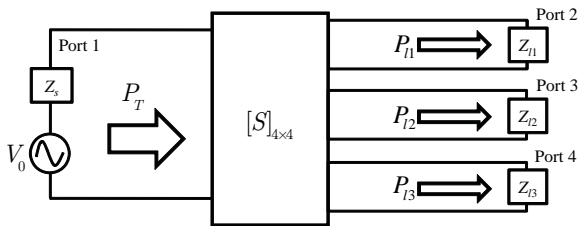


Fig. 4. 4-port network circuit for analysis model shown in Fig. 2.

because the maximum of PTE is obtained as the antenna is conjugate-matched with the port impedance of  $50 \Omega$  as stated in [6].

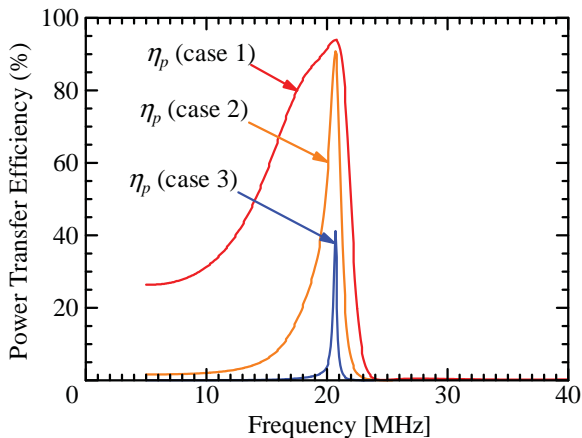


Fig. 5. The PTE of analysis model shown in Fig. 2 with different antenna positions shown in Table I, where antenna size is ( $l=30\text{cm}$ ).

#### IV. CONCLUSIONS

Multi-port scattering parameters were applied to the analysis of antennas in a WPT system corresponding to multi-user situations. The expression of PTE was defined in term of the multi-port scattering parameters. Some numerical simulations were shown to demonstrate it is easy to evaluate the PTE for various models of the antenna geometries, locations of

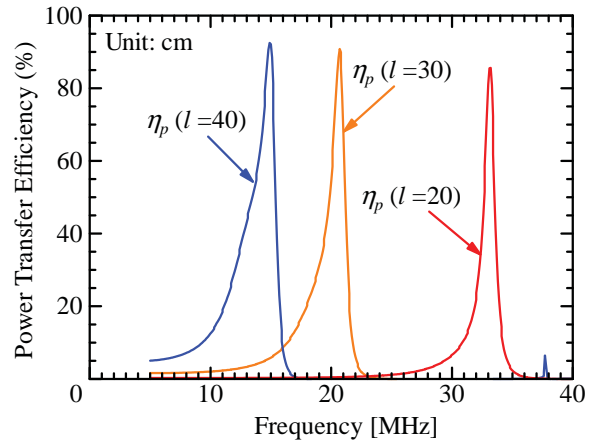


Fig. 6. The PTE of analysis model shown in Fig. 2 of Case 2 with different antenna size.

transmitting and receiving antennas in a multi-user WPT system by using the multi-port scattering parameters.

#### ACKNOWLEDGMENT

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