Efficiency Improvement with a Recursive Taylor Expansion of Bessel Functions for Layered Media Green's Function

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***Abstract*—This paper demonstrates the performance of a novel interpolation technique for computing the impedance matrix with a layered media Green's function (LMGF). The interpolation technique is based on a recursive Taylor expansion of a Bessel function of arbitrary order. Numerical simulation of microstrip array antennas embedded in a multi-layer medium is performed and the performance of the interpolation technique is demonstrated.**

I. Introduction

A microstrip antenna is well-known as one of the most practical antennas because it is low-profile and easy to fabricate. For example, many phased array antennas, frequency selective surfaces, and reflectarrays have been designed as microstrip structures [1]-[3]. In order to design the microstrip antenna, a numerical simulation technique such as method of moments (MoM) is useful [4]. The MoM with a layered media Green's function (LMGF) can deal with a multi-layer microstrip antenna efficiently [5]-[7]. One of the bottlenecks of the MoM with the LMGF is computation of a spectral integral, the so-called Sommerfeld integral.

In order to compute the Sommerfeld integral efficiently, various numerical technique have been proposed. A discrete complex image method (DCIM) [8]-[10] and polynomial interpolation technique [11]-[14] are popular approaches. Recently, our group has proposed a novel interpolation technique for the LMGF [15]. The proposed technique utilizes a recursive Taylor expansion of Bessel functions and is applicable to general multi-layer microstrip antennas. In this paper, the performance of the proposed interpolation technique is demonstrated for realistic problems.

II. Recursive Taylor Expansion

Fig. 1 shows two metallic microstrip dipole antennas embedded in a multi-layer medium. Self/mutual impedance between the microstrip dipoles can be obtained using the LMGF:

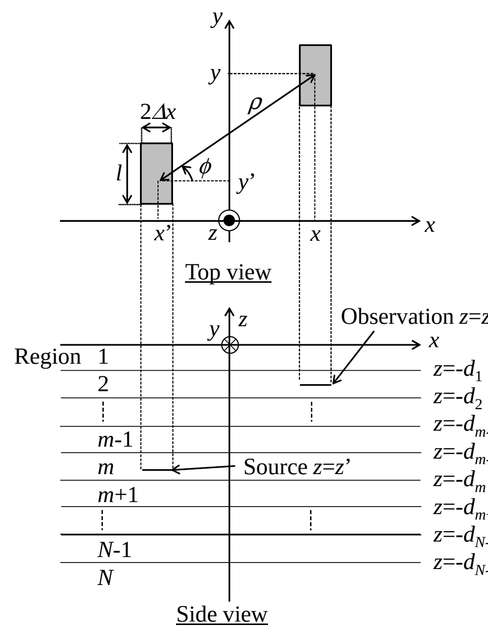

Fig. 1. Two planar microstrip segments in a thin-stratified medium.

$$Z_{ij}^{TE/TM} = j\omega\mu_0 \iint_S \mathbf{J}_y(x,y) \cdot \iint_{S'} \mathbf{J}_{y'}(x',y') G_{yy'}^{TE/TM}(\mathbf{r},\mathbf{r}')dx'dy'dxdy, \quad (1)$$

where $\mathbf{J}_y$ and $\mathbf{J}_{y'}$ are basis/testing functions, $G_{yy'}^{TE/TM}$ is the LMGF. As shown in [7], $G_{yy'}^{TE/TM}$ is expressed via the Sommerfeld integral which is computationally expensive. Eq. (1) includes multiple integrals for space variables and the computationally expensive Sommerfeld integral must be recomputed every time when the multiple integrals are performed. The spectral integral incorporates a Bessel function (see [15]). In order to reduce the computational cost of the LMGF, the

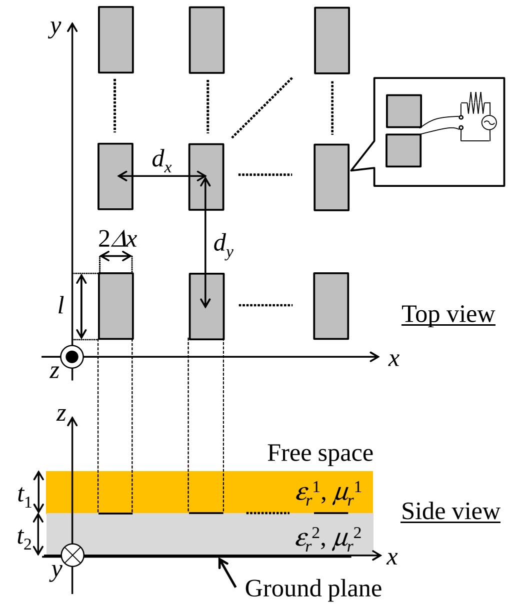

Fig. 2. Microstrip dipole array antenna embedded in a double layered medium backed by a ground plane.

Bessel functions are interpolated via Taylor expansion,

$$\begin{aligned} J_n(k_\rho \rho) &= J_n(k_\rho \rho_i) + \frac{\partial J_n(k_\rho \rho_i)}{\partial \rho}(\rho - \rho_i) \\ &+ \frac{1}{2}\frac{\partial^2 J_n(k_\rho \rho_i)}{\partial \rho^2}(\rho - \rho_i)^2 \\ &+ \frac{1}{6}\frac{\partial^3 J_n(k_\rho \rho_i)}{\partial \rho^3}(\rho - \rho_i)^3 + \cdots, \end{aligned} \tag{2}$$

where ρ_i is the ith sampling point. The coefficients of the polynomial in (2) can be obtained using the recursive property of Bessel functions. Therefore, the Sommerfeld integral is interpolated and the resultant number of spectral integrals to be computed is reduced using the proposed method. The proposed method is quite simple, straightforward, and easily applicable to general multi-layer problems.

III. Numerical Examples

A 20×20 mictosrtip dipole array antenna embedded in a double layer medium shown in Fig. 2 was numerically analyzed. The active resistance of the array elements is shown in Fig. 3. It is found that the proposed interpolation method is very accurate and efficient. The CPU time for computation of the impedance matrix entries is 291 and 17,431 sec. for the MoM with/without the proposed method, respectively. It is demonstrated that the proposed interpolation method reduces the CPU time for matrix fill by a factor of 60.

IV. Conclusions

In this paper, the performance of a novel interpolation method for the LMGF has been demonstrated. The interpolation method is applicable to general multi-layer problems

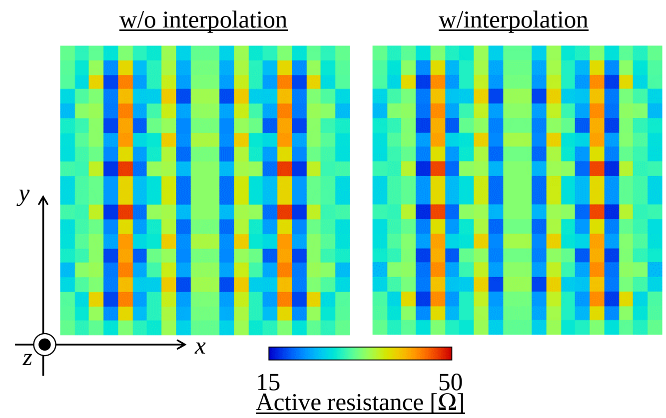

Fig. 3. Active resistance of a 20×20 array of microstrip dipole antennas (f =20 GHz, l =5 mm, $2\Delta x$ =1 mm, t_1 =2 mm, t_2 =1 mm, $d_x = d_y$ =7 mm, $\epsilon_r^1 = 2.6$, $\mu_r^1 = 1$, $\epsilon_r^2 = 2.15$, $\mu_r^2 = 1$).

because the LMGF is interpolated via recursive Taylor expansion. The CPU time and accuracy of the proposed method were demonstrated via numerical simulations with a factor of 60 reduction in the matrix fill time.

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