2-step Graph Coloring Algorithm for Cluster-wise Distributed MU-MIMO in Ultra-dense RAN

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***Abstract*—The ultra-dense radio access network with distributed MU-MIMO is considered as a promising approach to improve the coverage and the link capacity in 5G advanced systems. However, large-scale MU-MIMO requires prohibitively high computational complexity. Our previous work has proved that grouping neighborhood users/antennas into a number of clusters and performing cluster-wise MU-MIMO in parallel can alleviate the computational complexity problem, but the link capacity improvement is limited by the severe interference. In this paper, we propose a 2-step graph coloring algorithm that can eliminate both the inter-cell interferences and the inter-cluster interferences. The first step is to apply the graph coloring algorithm on the cell edge in order to reduce the inter-cell interferences. Once the color of the cell edge has been decided, the second step is to utilize the conditional graph coloring to the clusters within each BS cell. As a preliminary research, we focus on the second step and propose a Restricted Color Number (RCN) algorithm to mitigate the inter-cluster interferences. The computer simulation results show that our RCN algorithm can improve the link capacity compared with no coloring case.**

Keywords—2-Step graph coloring, interference coordination, ultra-dense RAN, distributed antennas, Delaunay triangulation, K-means algorithm

I. INTRODUCTION

Many countries have already started their 5G services. Because of the shortage of the available radio bandwidth, the ultra-dense radio access network (RAN) with distributed multi-user multi-input multi-output (MU-MIMO) is considered as a promising approach. However, a large-scale MU-MIMO requires prohibitively high computational complexity to deal with a large number of users and antennas in a base station (BS) coverage area (or cell), and therefore, finding a computationally efficient solution is necessary [1]. Our proposed solution is to group neighborhood users and antennas in each BS cell into clusters by K-means algorithm [2]. With clustering, the large-scale MU-MIMO is divided into several small-scale MU-MIMOs and the computational complexity will be reduced significantly. Furthermore, by reusing the same frequency simultaneously in clusters, the spectrum efficiency can be improved. Also, since the distance between user and antenna in a cluster is relatively short, the transmit power will be reduced, which in turn will results in the improvement in energy efficiency. However, the improvement of spectrum efficiency is limited by the inter-cluster interferences [3]. Furthermore, in multicell systems (shown in Fig. 1), the users in the cell edge areas suffer from the inter-cell interferences in addition to the inter-cluster interferences. Therefore, an effective interference coordination scheme that can mitigate both the inter-cluster interference and inter-cell interference is required for cluster-wise distributed MU-MIMO in ultra-dense RAN

Up till now, there have been many studies on the interference coordination. In dense small cell or femtocell systems, in order to mitigate the co-tier interference and cross-tier interference simultaneously, Yalan Zhao in [4] utilized the graph coloring algorithm to divide the dense small cells into different groups and then coordinated frequency resource to mitigate the severe co-tier interference, while at the meantime, coordinated the transmission power of macro base stations to alleviate the cross-tier interference; Similarly, Lu Chen in [5] also proposed to apply graph coloring algorithm on small cells to form color groups to solve the serious co-tier interference. Furthermore, Chen mentioned that the users at the edge area of small cells suffer worsen performance, and proposed to reschedule the frequency resources into two frequency bands as well as the users into two groups, one frequency band is assigned to edge users while another frequency band for center user. Through this way, the performance of edge users can be improved. Also, Qian Zhang in [6] utilized the graph coloring algorithm to group the least interfered cells together and reuse the same frequency to significantly mitigate the co-tier interference, while using orthogonal spectrum allocations to mitigate the cross-tier interference at the same time.

The interference coordination in ultra-dense RAN with distributed MIMO is an important research area. We observed that the inter-cluster interference inside each cell is similar to the co-tier interference, just as the inter-cell interference to the cross-tier interference. Based on this observation, we apply graph coloring algorithm twice to mitigate the inter-cell interference and inter-cluster interference together and propose a 2-step graph coloring-based interference coordination algorithm in ultra-dense RAN with distributed MU-MIMO. A structure of our proposed 2-step graph coloring algorithm is illustrated in Fig. 1. In this paper, we firstly introduce the overall idea of 2-step graph coloring algorithm, and then we will concentrate on the 2nd step that is designed to mitigate the inter-cluster interference in each BS cell (note that the 1st step of the 2-step graph coloring algorithm is left as our future study). In this paper, the 2nd step of the 2-step graph coloring algorithm is called the Restricted Color Number (RCN) graph coloring algorithm.

The rest of paper is organized as follows. In Sect. II, the concept of 2-step graph coloring based interference coordination is introduced together with the RCN algorithm . In Sect. III, the link capacity is evaluated by computer simulation. Finally, Sect. IV offers the conclusion and future research plan.

II. MU-MIMO IN ULTRA-DENSE RAN WITH DISTRIBUTED ANTENNAS

A. Concept of 2-step graph coloring algorithm

The first step is to apply the graph coloring algorithm to the cell edge in order to mitigate the inter-cell interference. Once the color of the cell edge is decided, the second step is to utilize the conditional graph coloring to the clusters in each BS cell to mitigate the inter-cluster interference (also shown in Fig. 1). As a preliminary study, this paper only focuses on the second step (see Sect. II. B) which is the RCN graph coloring algorithm. In order to achieve this goal, clusters need to be generated first, so in Sect. II. B, we also explained how to efficiently generate clusters in each BS coverage area.

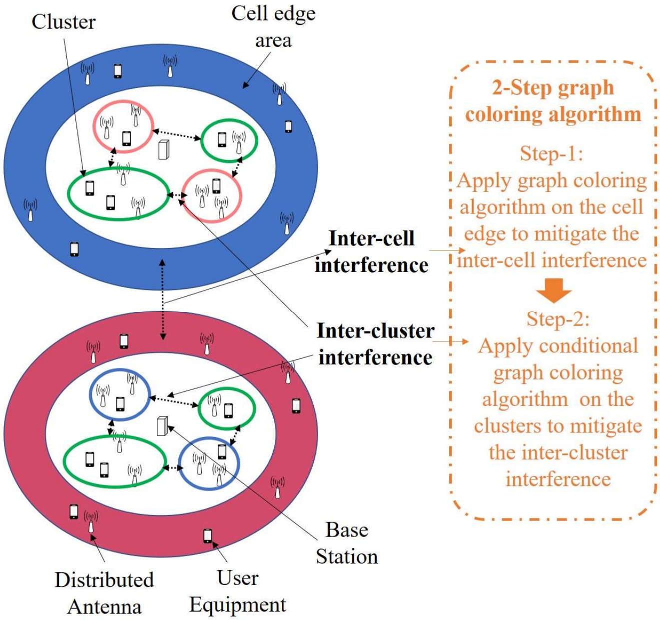


Fig. 1. 2-step graph coloring algorithm.

B. *RCN graph coloring algorithm*

a) Cluster formation

We assume a normalized square-shaped BS area of 1 by 1, in which 128 distributed antennas are randomly located and remain unchanged for all the trials in our Monte Carlo simulation in Sect. III . The number of users can be set between the number of clusters (at least each cluster has one user) and the number of antennas (to meet the requirement of Zero-Forcing (ZF) algorithm), and the users' locations changed every trial.

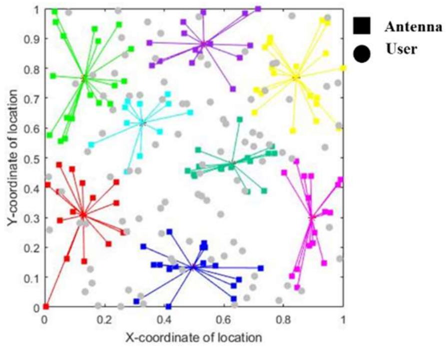
Firstly, the distributed antennas will be grouped into several clusters based on K-means algorithm according to the Euclidean distance. After that, the first layer of antenna clusters is formed and the cluster centroids is determined. Fig.2 (a) illustrates the cluster layout composed of 8 antenna clusters when the number of users is 96.

Secondly, based on the centroid, one-time K-means iteration is applied to form the second layer of user clusters (shown in Fig.2 (b)). With this method, we can guarantee to maximize the overlap region between the antenna clusters and user clusters.

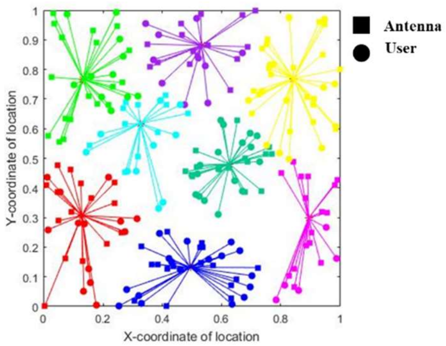
Because we assume the downlink transmission using ZF based cluster-wise distributed MU-MIMO, the number of users must be equal to or smaller than the number of antennas in each cluster. Thus a trading algorithm is also proposed to slightly rearrange some users in some clusters. After the K-means process mentioned above, thirdly, we need to calculate the number of users, U_k, and the number of antennas, A_k, in the *k*th cluster, and find the below:

- Vacancy clusters: clusters whose $A_k > U_k$;
- Flexible users: users in the clusters where $A_k < U_k$;

Fourthly, the nearest flexible users will be rearranged to those vacancy clusters one by one until the ZF precoding requirements is satisfied. The trading algorithm will only slightly rearrange the users' settlements, but the overall clustering results based on K-means will not be affected.



(a) the first layer of antenna clusters.



(b) the second layer of user clusters.

Fig. 2. Cluster generation.

b) Description of RCN graph coloring algorithm

Graph coloring algorithm has long been studied in cellular system, and in most cases it can be simplified as an undirected graph G= (V, E), in which V denotes vertices while E denotes edges. The vertices V represent different objects in different systems, such as the user equipment (UE) in large-scale phantom cells [7], the densely located small cells in femtocell system [6], and the clusters in D2D networks [8]. While the edges E always represent the interference relationship. If two "vertices" are interfered with each other and deserve to apply interference coordination, an "edge" should be applied between them. In each BS coverage area of the ultra-dense RAN with distributed antennas, since our clustering method makes sure that there will be no overlapping among the clusters, and the most severe interferences happen between the neighboring clusters, our situation can also be abstracted into an undirected graph where V denotes the centroid of clusters and E denotes the neighboring relationship. Furthermore, 'colors' means 'frequency' in this paper, in short, we are going to assign different frequency to neighboring clusters to avoid the most severe inter-cluster interference.

Our proposed RCN graph coloring algorithm consists of 4 Steps and the flow chart is shown in Fig. 3

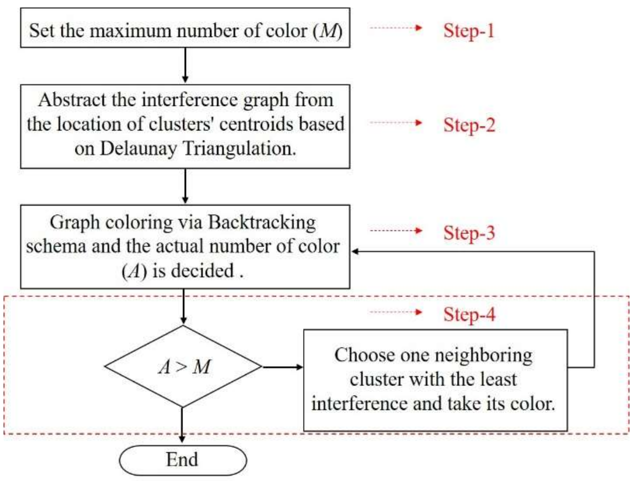


Fig. 3. RCN algorithm

Our RCN graph coloring algorithm restricts the maximum number of color (M), it doesn't require the number of color for every trials to be the same, on the contrary, it guarantees that the colors in each trial will not exceed M. With the existence of M, we can avoid dividing the available bandwidth into too many narrow parts. Therefore in step-1, we need to set the value of M in advance.

In step-2, an undirected interference graph needs to be abstracted from the location of clusters' centroids. In general, we need to predefine a threshold value as a criterion to judge whether the two vertices are interfered or not. However, there are no widely accepted standards on how to decide the threshold values wisely. Usually the threshold value is decided based on Euclidean distance [9] or interference level (SINR [10], pathloss [11], etc.), which relies a lot on the researchers' experience and deep understanding of the system. In this paper, we creatively introduce Delaunay Triangulation [11,12] from the view of computational geometry to decide the neighboring relationship automatically and avoid the use of threshold value.

A Delaunay triangulation for a given set of discrete vertices in a plane follows the restrictions that no vertex is inside the circumcircle of any triangles. The triangulation result for cluster formation of Fig.2 is shown in Fig.4, where the vertexes of each triangles represent the centroid of clusters. If there is a triangle's edge connecting two vertexes, these two clusters will be regarded as neighbors and cannot share the same color.

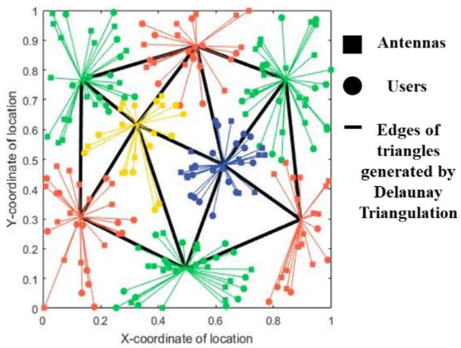


Fig. 4. Delaunay triangulation and RCN coloring result without restriction

Once the interference graph has been decided, the coloring outcome after Step-3 is determined. For the case in Fig. 2, the actual number of color (A) is 4 and the coloring results is also shown in Fig. 4 where different color represents the different frequency.

The essence of our proposed RCN algorithm is the restriction of maximum color number (M). Although more colors can guarantee better interference elimination results, more colors also indicate the narrower bandwidth each color group can share. From Shannon equation in (1), the sacrifice in bandwidth is a linear change while the elimination of interference is not a linear one. Therefore, more colors do not necessary results in higher link capacity and the restriction of M is needed.

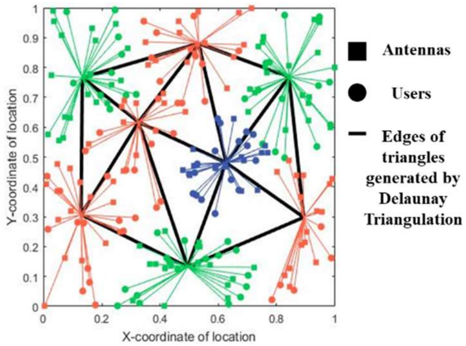
$$\begin{aligned} Capacity &= \frac{1}{Bandwidth}\log_2\left(1+SINR\right) \\ &\approx \frac{1}{Bandwidth}\log_2\left(SINR\right) \end{aligned} \tag{1}$$

Step-4 is the restriction process. Once $A \geq M$, some clusters need to share the same color with its neighbors. In order to minimize the inter-cluster interference, the least interfered cluster among the neighborhood should be chosen. The relative distance D_{ij} is introduced to help decide which cluster should be chosen. D_{ij} represents each cluster's interference level if we regard the cluster as a whole and neglect the distributed users and antennas inside. It's definition is shown in (2).

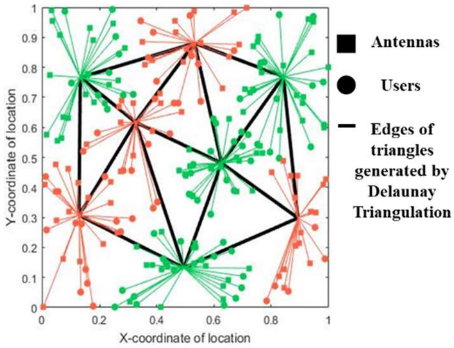
$$D_{ij} = \frac{d_{ij}^{-\gamma}}{\sum_{j=1,j\neq i}^{N} d_{ij}^{-\gamma}}, i,j = 1 \sim N \tag{2}$$

where d_{ij} denotes the distance from the cluster j's antenna center to cluster i's user center, N is the number of clusters, and γ is the pathloss exponent. Then the task for searching the least interfered clusters turn into the task searching for the least value in each row of the relative distance matrix D.

For the cluster formation of Fig.2, if we set M in Step-1 to be 2 or 3, then the restricted coloring results based on relative distance D_{ij} is shown in Fig. 5.



(a) 3 colors



(b) 2 colors

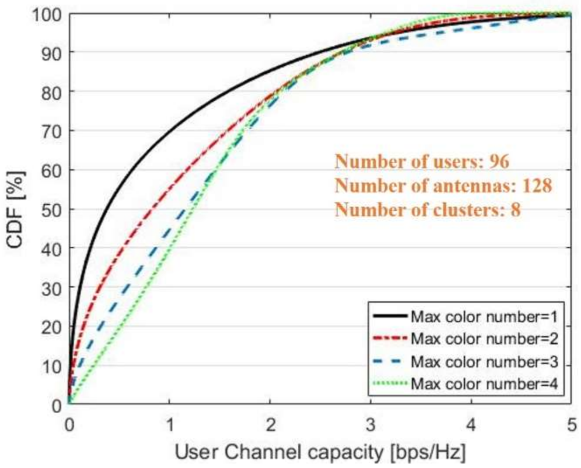
Fig. 5. RCN coloring results with restriction.

III. MONTE CARLO SIMULATION

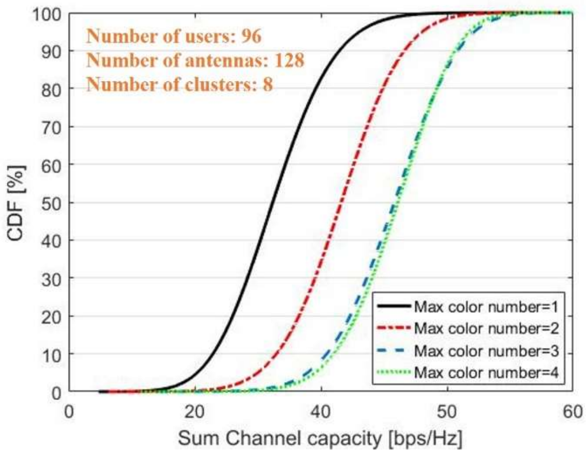
In this chapter, we evaluate the downlink sum capacity and user capacity when apply ZF-based cluster-wise distributed MU-MIMO. The transmit power for each user is set so that the received signal-to-noise ratio becomes 0dB when the distance between the transmitter and receiver is equal to the side length of square-shaped BS area. The pathloss exponent is set to be 3.5, and the shadowing standard deviation is 8dB. The fading type chose frequency- nonselective Rayleigh fading. All the CDF results are based on 1,000,000 trials assuming the stationary users communication for the certain set of antenna locations in Fig.2, while the user locations changed every trial.

In the following, the number of users, the number of antennas, and the number of clusters in each BS coverage area are represented by *U*, *A*, and *C*, respectively.

Figure 6 plots the cumulative distribution functions (CDFs) of the sum capacity and the user capacity for the case of 96 users, 128 antennas, and 8 clusters. The results reveal that overall, the graph coloring using 4 colors provides the highest capacity, followed by using 3 and 2 colors, but the advantage is narrowing for graph coloring using 3 and 4 colors. No color which means all clusters reuse the same whole bandwidth provides the lowest capacity. By increasing the number of colors from 2 to 3, the large improvement in capacity is obtained due to the elimination of the interference, but by increasing the number of colors from 3 to 4, the improvement is small because the elimination of the interference is offset by the sacrifice in bandwidth since the bandwidth assigned to each color group is reduced from 1/3 to 1/4 of the entire bandwidth. In a practical situation, more clusters will be formed and more colors will be needed. In order to avoid assigning a too narrow bandwidth to each color group, the restriction of maximum color number is of vital importance.



(a) User capacity

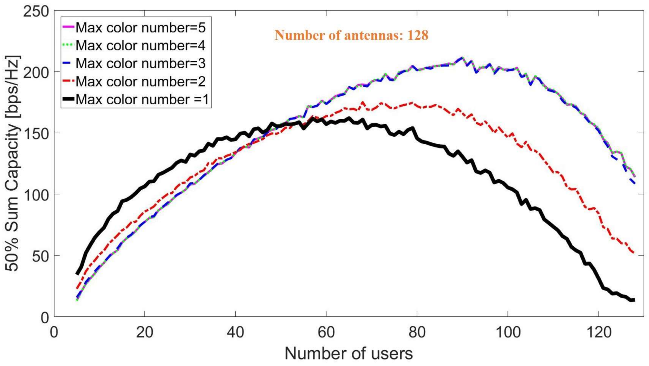


(b) Sum capacity

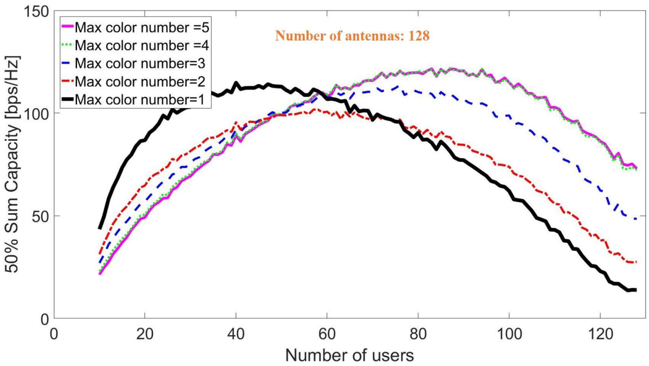
Fig. 6. CDF of sum capacity and user capacity for 96 users, 128 antennas, and 8 clusters.

Figure 7 plots the 50% sum capacity as a function of *U* when *C* equals 5, 10 and 15, and the value of *M* is from 2 to 5 (x% capacity is defined as the value of capacity below which the capacity falls at a probability of x%). From the simulation results, with the increase of *U*, the advantages of our proposed graph coloring algorithm (the colored lines) towards no coloring result (the black lines) is more obvious. The intersection of the coloring results and the no coloring one always happen when the number of users is near the midpoint. Since this is a preliminary research, we draw the rough conclusion that in practical engineering application, when *U* is more than half of *A*, our proposed RCN graph coloring algorithm has an advantage in promoting the sum capacity.

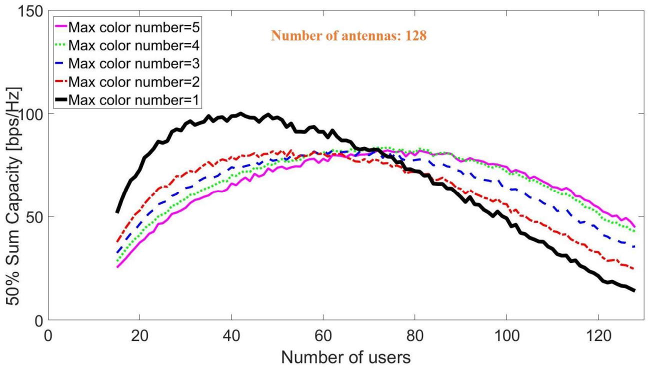
Figure 8 compares the 10% and 50% user capacity when *C* equals 5, 10 and 15. From the results below, our proposed RCN graph coloring algorithm can improve the 10% user capacity for all the cases of *U*. Also, the 50% user capacity will be improved when *U* is more than half of *A*.



(a) Number of clusters=5

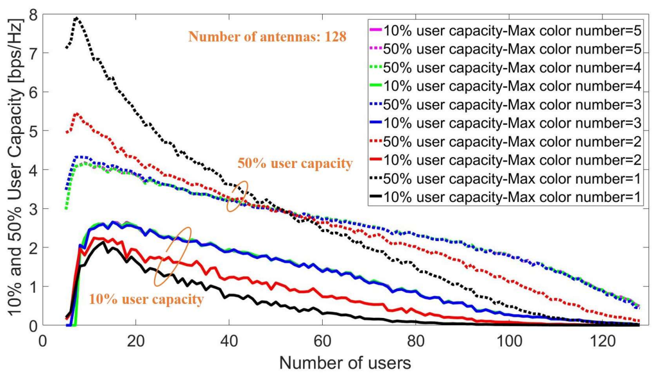


(b) Number of clusters=10

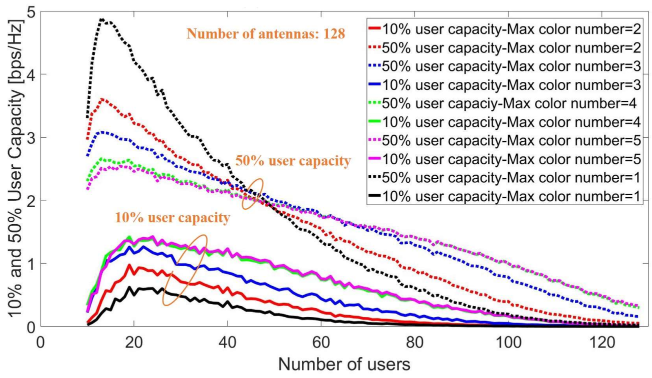


(c) Number of clusters=15

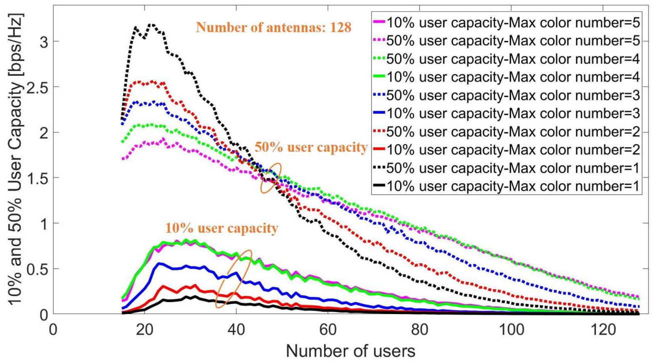
Fig. 7. Comparison of sum capacity when number of cluster is 5, 10 and 15



(a) Number of clusters=5



(b) Number of clusters=10



(c) Number of clusters=15

Fig. 8. Comparison of user capacity when the number of clusters is 5, 10 and 15.

IV. CONCLUSION

In this paper, we proposed the concept of 2-step graph coloring algorithm which can both eliminate the inter-cell interference and the inter-cluster interference at the same time. As a preliminary research, we mainly focused on the 2nd step, named Restricted Color Number (RCN) algorithm, to coordinate the interference inside each BS coverage area to mitigate the inter-cluster interference. In the RCN graph coloring algorithm, we introduced Delaunay triangulation from computational geometry to decide the neighboring relationship from the view of graphics and to avoid the use of threshold value. Also, our proposed algorithm can restrict the maximum number of color to avoid over-segmentation of valuable bandwidth.

From our analysis, we obtained the following findings:

- From the sum capacity point of view: we compared the 50% sum capacity, and concluded that the advantage of graph coloring continues to improve when the number of users is approaching the number of antennas. In practical engineering application, we recommend to apply the graph coloring algorithm when the number of users is more than half of the number of antennas.
- From the user capacity point of view: the 50% user capacity also be improved when the number of users is more than half of the number of antennas, but the 10% user capacity performed well in all the situations. So, we can draw the conclusion that our proposed graph coloring algorithm can further enhance the users' experience and satisfaction.

Our next step is to introduce the RCN graph coloring algorithm into the 2-step graph coloring algorithm for multicell system. After that, we also plan to introduce machine learning into our algorithm for further optimization.

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REFERENCES

[1] F. Adachi, R. Takahashi, and H. Matsuo, "Enhanced interference coordination and radio resource management for 5G advanced ultra-dense RAN," Proc. The 2020 IEEE 91st Vehicular Technology Conference (VTC2020-Spring): Technology Trials and Proof-of-Concept Activities for 5G Evolution & Beyond 5G 2020 (TPoC5GE 2020), 25 – 28 May, 2020.

[2] J. A. Hartigan and M. A. Wong, "Algorithm AS 136: A k-means clustering algorithm," J. Royal Statistical Society, Series C (Applied Statistics), Vol. 28, No. 1, pp. 100-108, 1979.

[3] S. Xia, C. Ge, Q. Chen, and F. Adachi, "K-means clustering and multi-user zero-forcing for ultra-dense RAN in 5G advanced system," IEICE Technical Report. vol. 119, no. 448, pp. 25-30, Mar. 2020.

[4] Y. Zhao, H. Xia, Z. Zeng and S. Wu, "Joint clustering-based resourse alllocation andpower control in dense small cell networks," Proc. IEEE/CIC ICCC 2015 Symposium on Wireless Communications Systems, Shenzhen, China, 2-4 Nov. 2015. DOI: 10.1109/ICCChina.2015.7448694.

[5] L. Chen, H. Xia, C. Feng and S.WU, "Clustering-based co-tier interference coordination in dense small cell networks," 2015 IEEE 26th International Symposium on Personal, Indoor and Mobile Radio Communications (PIMRC), Hong Kong, China, 30 Aug.- 2 Sept. 2015. DOI: 10.1109/PIMRC.2015.7343605.

[6] Q. Zhang, X. Zhu, and L. Wu, K. Sandrasegaran, "A coloring-based resource allocation for OFDMA femtocell networks," Proc. 2013 IEEE Wireless Communications and networking Conference (WCNC), Shanghai, China, 7-10 Apr. 2013. DOI: 10.1109/WCNC.2013.6554644.

[7] C. Zhao, X. Xu, and Z. Gao, L. Huang, "A coloring-based cluster resource allocation for ultra dense network," Proc. 2016 IEEE International Conference on Signal Processing, Communications and Computing (ICSPCC), Hong Kong, China, 5-8 Aug. 2016. DOI: 10.1109/ICSPCC.2016.7753602.

[8] J. Guo, L. Moalic, and J. Martin, A. Caminada, "Cluster resource assignment algorithm for device-to-device networks based on graph coloring," Proc. 13th International Wireless Communications and Mobile Computing Conference (IWCMC), Valencia, Spain, 26-30 June 2017. DOI: 10.1109/IWCMC.2017.7986540.

[9] S. Peng, E. Tong, and H. Jiang, Z. Pan, L. Nan, X. You, "An improved graph coloring based small cell discovery scheme in LTE hyper-dense networks," Proc. 2015 IEEE Wireless Communications and Networking Conference workshops (WCNCW), New Orleans, LA, USA, 9-12 Mar. 2015. DOI: 10.1109/WCNCW.2015.7122522.

[10] M. Liu, S. Tu, and C. Xiao, M. Waqas, S. Rehman, M. Aamir, "The allocation and reuse scheme of physical cell identifications based on maximum degree first coloring algorithm," IEEE Systems Journal, Vol. 14, No. 1, pp. 582-591, Mar. 2020. DOI: 10.1109/JSYST.2019.2911273.

[11] J. Park, N. Lee, and R. Health, "Cooperative base station coloring for pair-wise multi-cell coordination," IEEE Trans. Commun., Vol. 64, No. 1, pp. 402-415, Oct. 2015. DOI:10.1109/TCOMM.2015.2495355

[12] J. Keil, C. Gutwin, "Classes of graphs which approximate the complete euclidean graph," Discrete & Computational Geometry, Vol. 7, pp. 13–28, 1992. DOI: 10.1007/BF02187821.