Capacity-Fairness Tradeoff for Optimal Power Allocation in Cluster-wise MU-MIMO System

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***Abstract*— The cluster-wise multi-user multi-input multi-output (MU-MIMO), division of the ultra-dense network in the beyond fifth-generation (B5G) system into several small-scale MU-MIMO, is an effective way to reduce the computational complexity of the system operations. Reasonable power allocation is excepted for the improvement of system performance by adjusting the inter-cluster-interference. In this paper, we consider an optimal power allocation (OPA) method to realize the flexible tradeoff between sum capacity maximization and user fairness maximization while guaranteeing the minimum user capacity under the total transmit power constraint to meet the needs of different application scenarios. The simulation results verify that the proposed OPA method can effectively adjust the tradeoff between sum capacity and user fairness.**

Keywords—Beyond 5G, Distributed antenna system, MU-MIMO, User-clustering, Power allocation

I. Introduction

Since the mobile data traffic is ever-increasing, the massive multi-input multi-output (MIMO) becomes the key technology for mobile communication network densification in fifth-generation (5G) and beyond fifth-generation (B5G) systems to guarantee the quality of service (QoS) for a large number of users. Considering that the millimeter wave band is utilized to provide wider transmission bandwidth, compared with the centralized massive MIMO with large-scale array antennas co-located at a specific place, the distributed massive MIMO with a large number of antennas deployed in different positions can effectively avoid the blockage problem caused by the rectilinear propagation characteristics of the millimeter wave signals and provide a higher spectrum efficiency [1,2].

Additionally, the exorbitantly high computational complexity required for large-scale massive MIMO communication is unacceptable, so it is difficult to apply it to practice. Hence, we introduce the clustering method to solve the complexity problem by dividing the users and their associated antennas in each base station (BS) area (called the cell) into several clusters to perform cluster-wise multi-user MIMO (MU-MIMO) parallelly. However, when all clusters in the cell share the same radio resource, inter-cluster interference (ICI) limits the performance of the system. Therefore, in order to effectively mitigate the ICI, we consider an optimal power allocation (OPA) method. In our previous work [3], a sum capacity maximization-based OPA was proposed. Then, we realize that not only the sum capacity but also the user fairness is a significant system indicator for some application scenarios. And the improvement of sum capacity and user fairness are often contradictory. Consequently, in this paper, we propose an OPA method that can flexibly trade off the sum capacity maximization and user fairness maximization while guaranteeing the minimum user capacity requirement under the total transmit power constraint. Specifically, we utilize the well-known weighted sum method [4,5] in multi-objective optimization to realize the tradeoff between sum capacity and user fairness by introducing a tradeoff coefficient to combine the two objectives into one, so that it can be solved by the effective sequential quadratic programming (SQP) method [5,6] as a single-objective optimization.

The rest of this paper is organized as follows. In Section II, we introduce the system model of cluster-wise distributed MU-MIMO in a single cell. In Section III, we describe the proposed sum capacity and user fairness tradeoff-aware OPA method. In Section IV, the computer simulation results show the validity of proposed capacity-fairness tradeoff-aware OPA method. In the final Section V, we give some conclusions and implications.

II. System Model

For simplicity, in this paper, we consider a single cell in the cellular network to investigate the capacity and fairness tradeoff in cluster-wise distributed MU-MIMO system. U single-antenna users communicate with the BS through A distributed antennas which are connected to the BS via optical mobile fronthaul, and these users and antennas are exclusively divided into K clusters in order to reduce the computational complexity of the system to an acceptable level. Users and antennas are assumed to be randomly distributed in the cell, but in order to provide uniform QoS, a certain antenna distance (AS) is maintained between antennas so that antennas are approximately regularly distributed as much as possible.

Considering the mobility and aggregation of users, in the case of sufficient antenna deployment in the cell, we recommend a user-based clustering method based on their location information. After determining the user clustering, we further associate the adjacent antennas of each user into their cluster through the location relationship between users and antennas to perform the cluster-wise MU-MIMO communication. As a result of user-clustering and antenna association, the number of antennas in each cluster (A_k) is equal to or larger than the number of users in each cluster (U_k) to meet the MU-MIMO signal processing condition (i.e., A_k U_k). It is worth noting that assuming the same and limited multiplexing capacity in each cluster, we utilized the called modified K-means method which adds a number restriction of cluster members for each cluster to the classic K-means method [7,8].

An example of the cluster structure constructed inside a square-shaped single cell is shown in Fig. 1. Here, a normalized 1×1 square-shaped cell range is considered for simplicity and can be applied to any physical scale. We assume that U=64, A=2×U, AS=0.0625, and when assuming the number of users in each cluster shall not exceed 8, K=64/8=8. In Fig.1 triangles, circles, and cross marks indicate the coordinates of the user location, antenna location, and the centroid of the cluster, respectively. In addition, the solid and dashed lines in the figure show the cluster association of users and the antennas, respectively. We can see that the clusters are formed with the same number of users and that antennas are associated with user clusters according to the user cluster topology. The members of different clusters are represented by different colors, but it should be noted that the same radio resource is shared in all clusters in the cell, thereby producing the ICI.

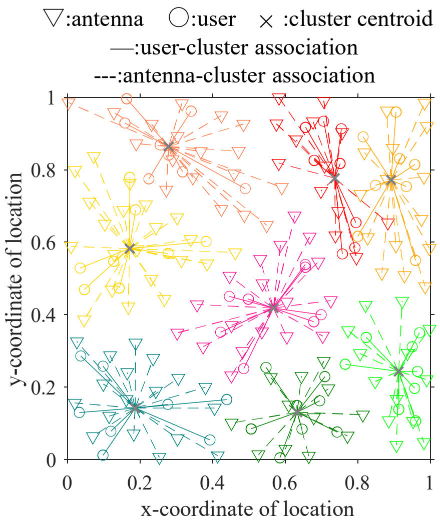


Fig. 1. An example of cluster structure after antenna assignment in a square-shaped single cell (U=64, A=2×U, AS=0.0625, K=8).

Subsequently, with the assumption of perfect channel state information at both user and BS side, zero-forcing (ZF) [9] based MU-MIMO transmission is implemented in each cluster separately to eliminate the inter-user-interference inside the cluster. The downlink channel matrix between the u_kth user in the kth cluster and all the antennas in the mth cluster is represented by $\mathbf{H}_{k,m}=\left[\mathbf{h}_{1_k,m}^T \ \cdots \ \mathbf{h}_{u_k,m}^T \ \cdots \ \mathbf{h}_{U_k,m}^T\right]^T$ and the precoding matrix for the kth cluster can be expressed as $\mathbf{W}_k=\mathbf{H}_{k,k}^{\dagger}=[\mathbf{w}_{1_k} \ \cdots \ \mathbf{w}_{u_k} \ \cdots \ \mathbf{w}_{U_k}]$, where $\mathbf{A}^{+}$ denotes the Moore–Penrose inverse of matrix $\mathbf{A}$. With transmit power for the u_kth user P_{u_k}, we describe the received signal of the u_kth user in the kth cluster as

$$y_{u_k}=\sqrt{\frac{P_{u_k}}{\left\|\mathbf{w}_{u_k}\right\|}}s_{u_k}+\sum_{\substack{m=0,\\ m\neq k}}^{K-1}\sum_{v_m=0}^{U_m-1}\mathbf{h}_{u_k,m}\mathbf{w}_{v_m}\sqrt{\frac{P_{v_m}}{\left\|\mathbf{w}_{v_m}\right\|}}s_{v_m}+n_{u_k}. \quad (1)$$

Then, by assuming the power spectral density of user signal and that of the additive white Gaussian noise (AWGN) have zero mean and unity variance, we further derive the capacity for the u_kth user in the kth cluster as

$$C_{u_k}=\begin{cases}\log_2\left(1+\dfrac{\dfrac{P_{u_k}}{\left\|\mathbf{w}_{u_k}\right\|^2}}{\sum\limits_{\substack{m=0,\\ m\neq k}}^{K-1}\sum\limits_{v_m=0}^{U_m-1}P_{v_m}\dfrac{\left|\mathbf{h}_{u_k,m}\mathbf{w}_{v_m}\right|^2}{\left\|\mathbf{w}_{v_m}\right\|^2}+1}\right), \text{downlink} \\ \log_2\left(1+\dfrac{P_{u_k}}{\sum\limits_{\substack{m=0,\\ m\neq k}}^{K-1}\sum\limits_{v_m=0}^{U_m-1}P_{v_m}\left|\mathbf{w}_{u_k}^T\mathbf{h}_{u_k,m}^T\right|^2+\left\|\mathbf{w}_{u_k}\right\|^2}\right), \text{uplink}\end{cases}, \quad (2)$$

where the capacity expression of the uplink according to the symmetry of the uplink and downlink. It should be noted that the transmission power P_{u_k} is independent in the uplink and downlink, and is the same only under the condition of equal power allocation (EPA).

III. Capacity-Fairness Tradeoff-aware OPA Method

In order to reasonably control the transmission power of each user in the system to suppress ICI and improve the performance of the system, we propose the capacity-fairness tradeoff-aware OPA method. Firstly, user capacity and sum capacity are often the criteria to evaluate the performance of the system. Therefore, in [3], we proposed an OPA strategy based on sum capacity maximization. However, we also note that in some application scenarios, user fairness is also an indicator that needs to be paid attention to. Therefore, this paper makes progress and expansion on the basis of [3] and considers the joint maximization of sum capacity and user fairness under the condition of ensuring the minimum user capacity demand. Specifically, our considered joint maximization of sum capacity and user fairness can be described as:

$$\max_{P_{u_k}}\sum_{k=0}^{K-1}\sum_{u_k=0}^{U_k-1}C_{u_k} \quad (3a)$$

$$\max_{P_{u_k}}\sqrt{\frac{\left(\sum\limits_{k=0}^{K-1}\sum\limits_{u_k=0}^{U_k-1}C_{u_k}\right)^2}{U_k\sum\limits_{k=0}^{K-1}\sum\limits_{u_k=0}^{U_k-1}C_{u_k}^2}} \quad (3b)$$

$$s.t.\ \sum_{k=0}^{K-1}\sum_{u_k=0}^{U_k-1}P_{u_k}=U\times P \quad (3c)$$

$$\forall C_{u_k}\geq C_{\min}. \quad (3d)$$

Here, the sum capacity maximization objective is formulated by (3a), and we utilize the well-known Jain's fairness index (JFI) [10] to represent the user fairness objective as in (3b). In addition, we fix the total transmit power of the system as the product of number of users (U) and a target transmit power for each user (P) in (3c). Then, in (3d), all user capacity must meet the restriction that are greater than or equal to the minimum user capacity ($C_{\min}$).

Because the multiple-objective optimal problem is difficult to be solve and the two objectives (3a) and (3b) are probably contradictory, we utilize the commonly used weighted sum method [4,5] to combine the two objectives into one and transform it into a single-objective optimization problem. In this way, by introducing the trade-off coefficient, the integrated objective function can be biased between the two original objectives. However, different from sum capacity, JFI is a unitless variable, and its value range is (0,1], so it is necessary to normalize the two objectives when weighting them or set an effective trade-off coefficient according to their value range difference. However, it is hard to achieve the above operations because the value range of sum capacity is determined by the user antenna position, clustering results, and channel state and it is hard to know exactly. Therefore, we equivalently convert JFI into the standard deviation of user capacity with the same base unit as the sum capacity. Consequently, the maximization of JFI in (3b) changes to the minimization of the standard deviation of the user capacity. Moreover, in order to effectively adjust the two objectives by the tradeoff coefficient α, we multiply the number of users in front of the standard deviation to make the value ranges of the two objective functions close. So, the capacity-fairness tradeoff-aware OPA problem can be modified as

$$\max_{P_{u_k}} \alpha \sum_{k=0}^{K-1}\sum_{u_k=0}^{U_k-1} C_{u_k} - (1-\alpha) U \sqrt{\frac{\sum_{k=0}^{K-1}\sum_{u_k=0}^{U_k-1} \left| C_{u_k} - \overline{C} \right|^2}{U}} \quad (4)$$

$$s.t.\ \text{(3c) and (3d)}$$

where $\alpha \in [0,1]$ is the tradeoff coefficient which represents the importance of objective, and $\overline{C}$ is the capacity averaged over all users.

So far, the capacity-fairness tradeoff-aware OPA problem of (4) is non-convex, so it is still difficult to solve directly. Therefore, we utilize the sequential quadratic programming (SQP) method [5,6] to approximate the optimal solution. As one of the most effective methods for the nonconvex constrained optimization, the essence of SQP is to gradually approach the optimal solution of the original problem by iterating the appropriate quadratic programming subproblem. Specifically, our formulated non-linear programming problem in (4) can be expressed in its general form as

$$\begin{aligned} &\min_{\mathbf{x}} f(\mathbf{x}) \quad . \\ \text{s.t.}\quad & c_i(\mathbf{x}) = 0, \quad i \in \mathrm{E} \\ & c_i(\mathbf{x}) \geq 0, \quad i \in \mathrm{I} \end{aligned} \quad (5)$$

By approximating the objective function and constraints in (5) linearly using Taylor expansion, the SQP method constructs the quadratic programming (QP) subproblem, at iteration t, given as

$$\begin{aligned} &\min_{d} \nabla f(\mathbf{x}_t)^T d + \frac{1}{2} d^T \mathbf{G}_t d \quad , \\ \text{s.t.}\ & c_i(\mathbf{x}_t) + \nabla c_i(\mathbf{x}_t)^T d = 0, \quad i \in \mathrm{E} \\ & c_i(\mathbf{x}_t) + \nabla c_i(\mathbf{x}_t)^T d \geq 0, \quad i \in \mathrm{I} \end{aligned} \quad (5)$$

where d is the search direction, t is the iteration index, $\mathbf{G}$ is the Hessian matrix (second-order partial derivatives matrix) of Lagrange function for (5)), and ∇ denotes the gradient. The SQP method solves the subproblems sequentially to approximate the optimal solution of the original problem. Our proposed SQP method is described in Algorithm 1.

Algorithm 1: SQP method

Initialization

Set iteration index t=0. Choose a starting point $\mathbf{x}_0$ and approximation Hessian $\mathbf{G}_0$.

Repeat

Solve the tth QP subproblem (6) to determine the search direction d_t

Determine the step size λ_t to update $\mathbf{x}_{t+1}=\mathbf{x}_t+\lambda_t$

Update the Hessian matrix $\mathbf{G}_{t+1}$, by

$$\mathbf{G}_{t+1} = \mathbf{G}_t + \frac{\mathbf{v}_t \mathbf{v}_t^T}{\mathbf{v}_t^T \mathbf{u}_t} - \frac{\mathbf{G}_t \mathbf{u}_t \mathbf{u}_t^T \mathbf{G}_t^T}{\mathbf{u}_t^T \mathbf{G}_t \mathbf{u}_t}, \quad (6)$$

Where $\mathbf{v}_t=\mathbf{x}_{t+1}-\mathbf{x}_t$ and $\mathbf{u}_t=\nabla L(\mathbf{x}_{t+1})-\nabla L(\mathbf{x}_t)$ with L denoting the Lagrange function.

t=t+1

Until stop criterion

IV. Numerical Results

In this section, we demonstrate and discuss the adjustability of the proposed weighted sum method based capacity-fairness tradeoff-aware OPA method. We carry out a Monte-Carlo simulation to obtain the cumulative distribution function (CDF) of user capacity, sum capacity, and user fairness by randomly changing the user location pattern 1000 times with a fixed randomly generated antenna location pattern. It should be emphasized that although we use standard deviation of user capacity to represent user fairness in the function composition of the proposed OPA for convenience, we still use JFI to measure user fairness when calculating CDF results.

For each user location pattern, user-clustering and cluster-antenna association are carried out. After the channel decided, the user capacity is computed using (2) to obtain the sum capacity and user fairness. The MIMO channel is characterized by path loss, log-normal shadowing loss, and Rayleigh fading. In our simulation, a quasi-static channel condition is considered which means we randomly generate shadowing and fading channels once when the user location pattern changes.

As a benchmark, we also calculated the results of equal power allocation (EPA) case to evaluate the performance of the proposed OPA. For EPA case, users are equally assigned the transmit power. The transmit power for each user is represented by the normalized transmit signal-to-noise ratio (SNR) which is defined as the received SNR when the transmitter-receiver distance is equal to the side length of the normalized 1×1 square-shaped area. In addition, we set the initial state for iteration in SQP method to the EPA state and if the capacity-fairness tradeoff-aware OPA cannot find a feasible solution the EPA state is adopted for the capacity calculation.

TABLE I. Simulation Setting

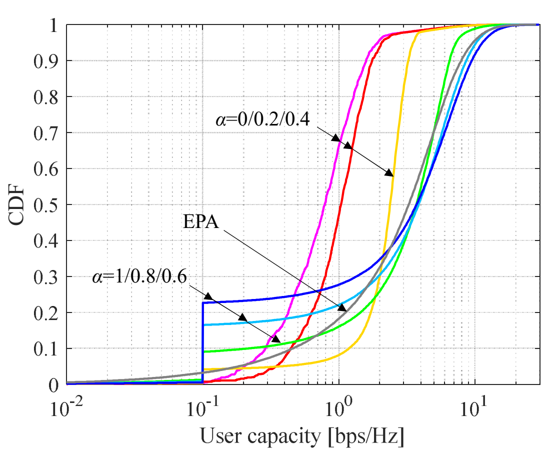
Parameter	Value/State
Number of distributed antennas (A)	128
Number of users (U)	64
Number of clusters (K)	8
Number of times of user location generations	1000
Path loss exponent	3.5
Log-normal shadowing standard deviation [dB]	8
Fading type	Rayleigh

Transmit SNR per user (P) [dB]	0
Minimum user capacity ($C_{\min}$) [bps/Hz]	0.1
Starting point of SQP method	EPA state
Tradeoff coefficient (α)	0/0.2/0.4/0.6/0.8/1

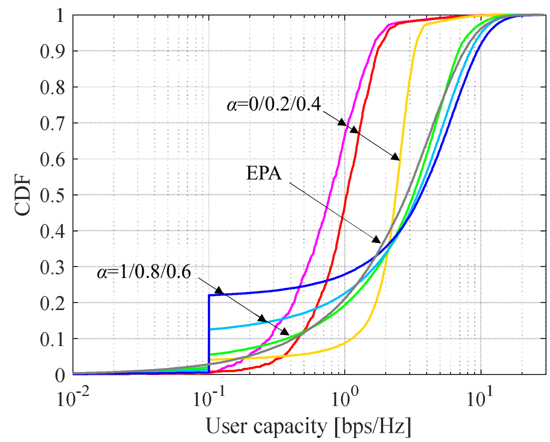
The CDF results of user capacity for both downlink and uplink with different values of α in the proposed OPA are presented in Fig. 2, where the results of the EPA case are illustrated by gray lines as a reference. Firstly, we can see that compared with the EPA case, the proposed OPA commendably ensures the minimum user capacity under any value of α. Moreover, the CDF curve changes obviously by changing α from 0 to 1, which suggests that adjusting α can effectively bias the transmit power allocation toward the capacity objective or the fairness objective. In other words, our objective function design based on the weighted sum method is successful.

In precise, we note that when α is set as 1, i.e., the objective function only considers sum capacity maximization, the probability of user capacity restricted to $C_{\min}$ is highest (over 20%), and achievable maximum user capacity is much higher than that of EPA case on the other hand. This happens because the transmit power of users with a poor channel condition tends to be allocated to the users with a better channel condition to further increase their capacity to maximize the sum capacity. This is the same result as the previous single-objective OPA in [3].

On the other hand, when α reduces, the transmit power allocation is biased towards high user fairness. Specifically, the probability of user capacity becoming equal to $C_{\min}$ can be gradually reduced by reducing α. Moreover, the probability of the capacity exceeding a certain high capacity and the achievable maximum user capacity can be both reduced. As a consequence, higher user fairness is obtained with a smaller α. As a result, another extreme case (α=0) meets the user fairness maximization objective.



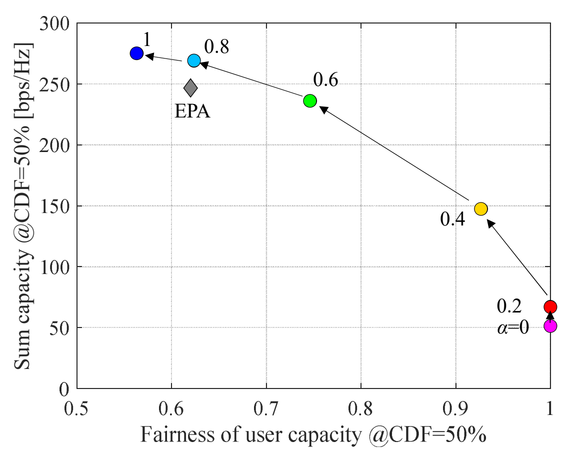
(a) Downlink



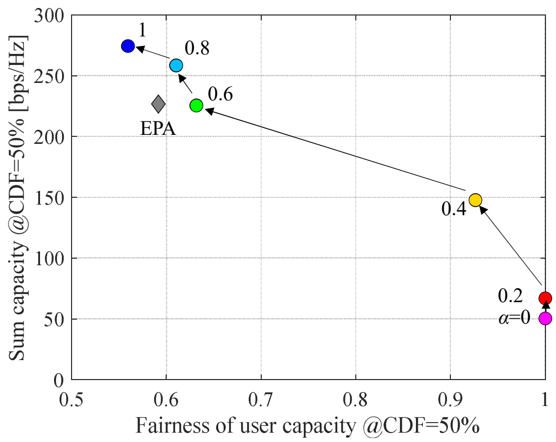
(b) Uplink

Fig. 2. CDF comparison of user capacity with different values of α in capacity-fairness tradeoff-aware OPA.

Then, we investigate the impact of α on two indicators (sum capacity and user fairness) intuitively as illustrated in Fig. 3. Here, the horizontal ordinate and vertical ordinate represent the sum capacity at CDF=50% and the user fairness at CDF=50%, respectively. From Fig. 3, we can clearly see that adjusting α can effectively bias the transmit power allocation toward the sum capacity objective or the user fairness objective. When α is equal to 0, capacity-fairness tradeoff-aware OPA considers the user fairness maximization only and the sum capacity becomes lowest. On the other hand, by increasing α, the user fairness decreases gradually, and the sum capacity increases gradually. In addition, we can also see from Fig. 3 that when α is equal to 0.8, both the sum capacity and the user fairness are improved compared to the EPA case.



(a) Downlink



(b) Uplink

Fig. 3. Relationship between sum capacity and user fairness with different values of α in capacity-fairness tradeoff-aware OPA.

It can be summarized from Figs. 2 and 3 that the proposed capacity-fairness tradeoff-aware OPA method based on the weighted sum method can flexibly trade off the sum capacity maximization and the user fairness maximization by changing the value of α while guaranteeing the minimum user capacity, which indicates that the proposed capacity-fairness tradeoff-aware OPA can be applied to many practical application scenarios with a variety of QoS.

V. Conclusions

In this paper, we proposed a capacity-fairness tradeoff-aware OPA method for the cluster-wise distributed MU-MIMO system to meet the needs of different application scenarios in the B5G systems. We formulated the proposed capacity-fairness tradeoff-aware OPA with the minimum user capacity guarantee and limited total transmit power constraint by combining two contradictory objectives into one based on the weighted sum method. Then, we introduced the effective SQP algorithm to solve the proposed OPA problem.

From the Monte-Carlo simulation, we demonstrated that the sum capacity and the user fairness can be flexibly traded off while guaranteeing the minimum user capacity by adjusting the tradeoff coefficient α in the proposed OPA. The simulation results also indicated that an appropriate selection of α can improve the sum capacity and user fairness simultaneously compared with the EPA case.

In fact, ensuring the minimum capacity requirement of users is often a key indicator. However, meeting the minimum capacity requirement is closely related to user distribution, clustering results, and channel status. In a real environment, guaranteeing the minimum capacity may not be possible. How to effectively avoid such a situation is left as our future work. In this paper, we considered a square-shaped single-cell. In a cellular system, the inter-cell interference is produced. An extension of our work to the cellular system is also a remaining issue.

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