# Enhancement of Power Transfer Efficiency of MIMO-WPT System by Optimal Load Impedance

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Abstract — In this paper, power transfer efficiency of a multi input multi output wireless power transfer (MIMO-WPT) system is enhanced by the optimal load impedance. The optimal current and the optimal load impedance of the MIMO-WPT system are obtained from Z-parameters of the system. It is demonstrated that power transfer efficiency of the MIMO-WPT system with the optimal load impedance outperforms that with 50  $\Omega$  load impedance.

Index Terms — Impedance parameters, MIMO-WPT, Optimal load impedance, Rayleigh quotient, Scattering parameters.

#### I. INTRODUCTION

Recently, the maximum power transfer efficiency of the MIMO-WPT system has been studied [1]-[3]. Wiedmann and Weber have discussed the maximum power transfer efficiency of the MIMO-WPT system using a theory of Rayleigh quotient [1]. Yuan et al., has formulated a generalized Rayleigh quotient problem of the MIMO-WPT system [2][3]. According to the generalized Rayleigh quotient problem formulated in [2], the optimal current and load impedance of the MIMO-WPT system for maximizing the power transfer efficiency have been explicitly obtained. On the other hand, Wen et al., has also studied the maximum power transfer efficiency of the MIMO-WPT system based on a theory of Rayleigh quotient [4]-[6]. Their works are similar to those of the former two groups but discussion on the optimum load impedance for the MIMO-WPT system is absent. Studies on the design method of impedance matching circuits for WPT systems [7][8] have already been reported and impact of impedance matching on efficiency of the WPT systems has been demonstrated in these studies.

In this paper, power transfer efficiency of the MIMO-WPT system with the optimal load impedance is clarified. The optimal load impedance is obtained from the theory of the Rayleigh quotient proposed in [2]. It is demonstrated that the maximum power transfer efficiency of the MIMO-WPT system with the optimal load impedance outperforms that with the 50  $\Omega$  resistance.

# II. THE MAXIMUM POWER TRANSMISSION EFFICIENCY OF MIMO-WPT SYSTEM

Let us consider a MIMO-WPT system with M elements of transmitting antennas and N elements of receiving antennas.

The MIMO-WPT system is represented by M + N ports networks using impedance parameters. Input/Output powers of the network can be obtained using the impedance parameters as follows.

$$P_{in} = \frac{1}{2} \operatorname{Re}([I_t]^H[V_t]) = \frac{1}{4} [I]^H[B][I]$$
  
$$= \frac{1}{4} \begin{bmatrix} [I_t] \\ [I_r] \end{bmatrix}^H \begin{bmatrix} [Z_{tt}] + [Z_{tt}]^H & [Z_{tr}] \\ [Z_{tr}]^H & [0] \end{bmatrix} \begin{bmatrix} [I_t] \\ [I_r] \end{bmatrix}, (1)$$
  
$$P_{out} = -\frac{1}{2} \operatorname{Re}([I_r]^H[V_r]) = -\frac{1}{4} [I]^H[A][I]$$
  
$$= -\frac{1}{4} \begin{bmatrix} [I_t] \\ [I_r] \end{bmatrix}^H \begin{bmatrix} 0 & [Z_{rt}]^H \\ [Z_{rt}] & [Z_{rr}] + [Z_{rr}]^H \end{bmatrix} \begin{bmatrix} [I_t] \\ [I_r] \end{bmatrix}, (2)$$

where,  $P_{in}$  is the input power and  $P_{out}$  is the output power of the network. The power transfer efficiency of the MIMO-WPT system can be defined in the same manner with reference [2] as follows.

$$\eta = \frac{P_{out}}{P_{in}} = -\frac{[I]^H [A][I]}{[I]^H [B][I]},$$
(3)

where, superscript H means complex conjugate transpose of a matrix. (3) is in the form of a generalized Rayleigh quotient. According to (3), the maximum power transfer efficiency of MIMO-WPT system can be obtained as the maximum eigenvalue of (3). An eigenvector corresponding to the maximum eigenvalue is the optimal current vector [I]. Owing to the optimal current, the optimal load is obtained as follows.

$$Z_{l\,i}^{opt} = \left(\frac{V_{r\,i}^{opt}}{I_{r\,i}^{opt}}\right)^* \quad (i = 1, \cdots, N), \tag{4}$$

where,  $I_{r\,i}^{opt}$  is the optimal current at the *i* th receiving antenna port and  $V_{r\,i}^{opt}$  is the resultant voltage at the *i* th receiving antenna port obtained from the product of impedance parameters and the optimal current.

## III. NUMERICAL ANALYSIS RESULTS

Effect of the optimal load impedance on the power transfer efficiency of the MIMO-WPT system is clarified here. Fig. 1 shows a MIMO-WPT system. A planar array of  $4 \times 4$  dipole antennas is a transmitting antenna. Three dipole antennas are receiving antenna and are positioned at  $S(x_1, y_1, z_1) =$  $(-s - d\cos(\pi/6)/\sqrt{2}, -s - d\cos(\pi/6)/\sqrt{2}, d\sin(\pi/6))$ ,  $S(x_2, y_2, z_2) = (s + d\cos(\pi/6), 0, z_1)$  and  $S(x_3, y_3, z_3) =$  $(x_1, -y_1, z_1)$  where *s* is the array spacing of the transmitting array antennas. The scattering and impedance parameters of the MIMO-WPT antenna are obtained using the commercial simulator software FEKO [9]. All the dipole antennas are half-wavelength dipole antennas, and the array spacing *s* of the transmitting array antennas is  $0.65\lambda_0$ .



Fig. 1. Numerical Analysis Model 16×3MIMO-WPT

The maximum power transfer efficiencies of the MIMO-WPT system with difference load impedance are shown in Fig. 2. In order to demonstrate the performance of the propose method clearly, boundaries among reactive near-field region, Fresnel region and far-field region are shown in Fig. 2. It is found that the maximum power transfer efficiency of the MIMO-WPT system with the optimal load impedance outperforms that with the 50  $\Omega$  load. The difference between the efficiencies comes from difference of the load impedance. The optimal load at each receiving antenna is shown in Fig. 3. In far field region, the optimal load converges to the conjugate complex number of the impedance of halfwavelength dipole antenna and satisfies the impedance matching condition. On the other hand, in the near field region, the optimal load is not coincident with 50  $\Omega$  load because of the effect of mutual coupling between antennas. Therefore, it can be said that the maximum power transfer efficiency of the MIMO-WPT system is enhanced when the receiving antenna is terminated by the optimal load impedance.



Fig. 2. Simulated Power Transfer Efficiency 16×3MIMO-WPT.



Fig. 3. Simulated Optimal Load at Receiving Antenna.

### IV. CONCLUSION

In this paper, power transfer efficiency of the MIMO-WPT system with the optimal load impedance has been clarified. The optimal load impedance was obtained from the theory of the Rayleigh quotient. It has been demonstrated that the maximum power transfer efficiency of the MIMO-WPT system with the optimal load impedance outperforms that with the 50  $\Omega$  resistance.

#### ACKNOWLEDGEMENT

This work was financially supported by JSPS KAKENHI Grant Number 18K13736 and 18K04116. Part of this work was carried out under the Cooperative Research Project Program of the Research Institute of Electrical Communication, Tohoku University.

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