Efficient Modeling of Two-Dimensional Infinite Periodic Structures and Its Application to Method of Moments

Keisuke Konno and Qiang Chen Department of Communications Engineering, Graduate School of Engineering, Tohoku University, Sendai, Japan {keisuke.konno.b5, qiang.chen.a5}@tohoku.ac.jp

Abstract—A novel modeling method of two-dimensional infinite periodic structures is introduced, and they are modeled as infinite surfaces whose reflection coefficients are numerically known. In the similar manner as a layered media Green's function (LMGF), a reduced form of Green's function for the two-dimensional infinite periodic structures is obtained. Self/mutual coupling between source and observation point is formulated via method of moments (MoM). Numerical simulation of an antenna over frequency selective surfaces (FSSs) is performed, and it is found that the proposed modeling method works well for including contribution of the FSSs to the numerical results.

Keywords—Periodic structure, Reflection coefficient, Green's function, Method of moments, Frequency selective surface

I. INTRODUCTION

Periodic structures have been widely used for designing practical antennas or scatterers, such as array antennas, FSSs, radio wave absorbers, and reflectarrays [1-3]. For example, array antennas for wireless power transfer systems and wireless communications have been designed as periodic structures[4]-[9]. Radio wave absorbers for radio frequency identification systems have been developed as periodic structures [10, 11]. Reflectarrays for wireless communication systems have been designed as periodic structures [12-14]. Diagnosis technologies for finding defective elements or unintentional radiation sources using eigenmode currents of periodic array antennas have been developed so far [15-17].

On the other hand, numerical analysis methods are necessary for designing the periodic structures. MoM is wellknown as one of the efficient numerical analysis methods for periodic structures [18]. During the numerical analysis of the periodic structures using MoM, two different options are available for their modeling, i.e. modeling as finite ones or infinite ones. Numerical analysis of the finite periodic structures using the MoM requires large computational cost because their size is usually larger than wavelength. Therefore, reduction of computational cost using fast MoM such as iterative solvers or direct solvers is necessary [19-22]. On the other hand, numerical analysis of the infinite periodic



Infinite surface whose reflection coefficients $\Gamma^{\text{TE/TM}}(\theta, \phi)$ are known

Fig. 1 Concept of the presented method.

structures using the MoM only requires small computational cost because the infinite periodic structures can be modeled as so-called unit cell via a periodic boundary condition (PBC) [23-26].

It is well-known that periodic structures enhance the performance of antennas. For example, an artificial magnetic conductor has been used to design low-profile antennas [27]. Instead of ground planes, bandstop FSSs have been introduced and multiband reflectarrays have been developed [28]. Although the periodic structures can enhance performance of antennas, numerical analysis of antennas with periodic structures is cumbersome.

In this paper, a novel modeling method of two-dimensional infinite periodic structures demonstrated in [29] is presented. The presented method models the infinite periodic structures as infinite surfaces via reflection coefficients. The reflection coefficients of the infinite periodic structures are easily available using PBC. Once the reflection coefficients are obtained, response from the infinite periodic structures can be

979-8-3503-5558-1/24/\$31.00 ©2024 IEEE

formulated by a form of Green's function. Self/mutual impedance expression between source and observation points is obtained by MoM formalism. Numerical analysis is performed, and it is demonstrated that the presented method enables to model the infinite periodic structures accurately.

II. MODELING OF INFINITE PERIODIC STRUCTURES

Fig. 1 shows concept of the modeling of the infinite periodic structures via the reflection coefficients. The presented method models the infinite periodic structures as a surface whose reflection coefficients are numerically known. The reflection coefficients of the infinite periodic structures are obtained using MoM with PBC. In the similar manner as the LMGF, electromagnetic response between source and observation points can be described as summation of freespace Green's function corresponding to direct wave and TE/TM components of dyadic Green's function corresponding to reflection wave [30-33]. Once the electromagnetic response between source and observation points are described, self/mutual impedance between them can also be described using MoM. According to the presented method, mesh-free modeling of the infinite periodic structures is available. This point is a big advantage of the presented method. Rigorous description on the presented method is omitted here due to the limitation of the pages but is found in a previous publication [29].

III. NUMERICAL ANALYSIS

Here, performance of the presented method is demonstrated via a numerical example. Numerical analysis of a rectangular loop antenna over FSS composed of circular loop elements shown in Fig. 2 is performed.

Directivity of the rectangular loop antenna over the FSS is shown in Fig. 3. As a reference, Fig. 3 includes the directivity of the rectangular loop antenna over finite FSS composed of 7 x 7 circular loop elements. It is found that directivity of the rectangular loop antenna over the infinite FSS obtained using the presented method agrees well with that over the finite FSS. At this frequency band, it has been confirmed that the FSS demonstrates similar reflection performance as a ground plane [29]. Therefore, direct wave from the rectangular loop antenna and reflection wave from the FSS are in-phase at broadside direction. As a result, directivity of the rectangular loop antenna over the FSS becomes maximum at the broadside direction.

TABLE I shows CPU time for numerical analysis. The CPU time corresponds to 61 points of frequency response from 100 MHz to 400 MHz. It is found that CPU time of the presented method is much shorter than that of Full-wave analysis. As have been mentioned earlier, the presented method enables mesh-free modeling of the FSS and the total number of unknowns is only 44. On the other hand, full-wave analysis (i.e. conventional MoM) requires mesh modeling of the FSS and the total number of unknowns is 2249. Such difference of the total number of unknowns results in significant difference of the CPU time.



Fig. 2 Rectangular loop antenna over circular loop FSS



TABLE I.

Method	CPU time [sec.]
Full-wave	21454
Presented	210

CPU TIME

IV. CONCLUSIONS

In this paper, a novel modeling method of two-dimensional infinite periodic structures was presented. The presented method enables mesh-free modeling of the two-dimensional infinite periodic structures using their reflection coefficients. Numerical simulation was performed, and it was demonstrated that the presented method reduces computational cost without degrading accuracy.

ACKNOWLEDGMENT

This research was partly supported by the Ministry of Internal Affairs and Communications in Japan (JPJ000254). Discussions with the members of the Cooperative Research Project Program of the Research Institute of Electrical Communication, Tohoku University, were helpful for this work.

REFERENCES

- R. C. Hansen, Phased Array Antennas, Sec. 8.3, John Wiley & Sons, 1998.
- [2] B. A. Munk, Frequency Selective Surfaces: Theory and Design, New York, NY, USA: Wiley, 2000.
- [3] J. Huang and J. A. Encinar, Reflectarray Antennas, John Wiley and Sons, 2008.
- [4] N. Shinohara and H. Matsumoto, "Experimental study of large rectenna array for microwave energy transmission," IEEE Trans. Microw. Theory Tech., vol. 46, no. 3, pp. 261-268, March 1998.
- [5] Y. Konishi, "Phased array antennas," IEICE Trans. Commun., vol. E86-B, no. 3, pp. 954-967, March 2003.
- [6] K. Konno, Q. Chen, K. Sawaya, and T. Sezai, "Analysis of huge-scale periodic array antenna using impedance extension method," IEICE Trans. Commun., vol. E92-B, no. 12, pp.3869-3874, Dec. 2009.
- [7] K. Konno, Q. Chen, K. Sawaya, and T. Sezai, "Statistical analysis of huge-scale periodic array antenna including randomly distributed faulty elements," IEICE Trans. Electron., vol. E94-C, no.10, pp.1611-1617, Oct. 2011.
- [8] K. Konno, Q. Yuan, and Q. Chen, "Ninja Array Antenna: Novel Approach for Low Backscattering Phased Array Antenna," IET Microw. Antennas Propag., vol. 12, no. 3, pp.346-353, 2018.
- [9] K. Konno, K. Morita, Q. Chen, and Q. Yuan, "Experimental Study of Ninja Array Antenna Composed of Yagi-Uda Antennas," IEICE Commun. Express, vol. 8, no. 12, pp. 554-559, Dec. 2019.
- [10] S. Yagitani, K. Katsuda, M. Nojima, Y. Yoshimura, H. Sugiura, "Imaging radio-frequency power distributions by an EBG absorber," IEICE Trans. Commun., vol. E94-B, no. 8, pp. 2306-2315, Aug. 2011.
- [11] Y. Okano, S. Ogino and K. Ishikawa, "Development of optically transparent ultrathin microwave absorber for ultrahigh-frequency RF identification system," IEEE Trans. Microw. Theory Tech., vol. 60, no. 8, pp. 2456-2464, Aug. 2012.
- [12] L. Li, Q. Chen, Q. Yuan, K. Sawaya, T. Maruyama, T. Furuno, and S. Uebayashi, "Frequency selective reflectarray using crossed-dipole elements with square loops for wireless communication applications," IEEE Trans. Antennas Propag., vol. 59, no. 1, pp. 89-99, Jan. 2011.
- [13] T. Smith, U. Gothelf, O. S. Kim and O. Breinbjerg, "An FSS-backed 20/30 GHz circularly polarized reflectarray for a shared aperture L- and Ka-band satellite communication antenna," IEEE Trans. Antennas Propag., vol. 62, no. 2, pp. 661-668, Feb. 2014.
- [14] H. Kamoda, T. Iwasaki, J. Tsumochi, T. Kuki, and O. Hashimoto, "60-GHz electronically reconfigurable large reflectarray using single-bit phase shifters," IEEE Trans. Antennas Propag., vol. 59, no. 7, pp.2524-2531, July 2011.
- [15] K. Konno, S. Asano, T. Umenai, and Q. Chen, "Diagnosis of Array Antennas Using Eigenmode Currents and Near-Field Data," IEEE Trans. Antennas Propag., vol. 66, no. 11, pp.5982-5989, Nov. 2018.
- [16] X. Wang, K. Konno, and Q. Chen, "Diagnosis of Array Antennas Based on Phaseless Near-Field Data Using Artificial Neural Network," IEEE Trans. Antennas Propag., vol. 69, no. 7, pp. 3840-3848, July 2021.
- [17] K. Mochiki, K. Konno, and Q. Chen, "Estimation of Equivalent Current Distribution Using Fourier Transform and Eigenmode Currents," IEEE Trans. Electromagn. Compat., vol. 64, no. 5, pp. 1380-1390, Oct. 2022.

- [18] R. F. Harrington, Field Computation by Moment Methods, Macmillan, New York, 1968.
- [19] K. Konno, Q. Chen, and K. Sawaya, "Quantitative evaluation for computational cost of CG-FMM on typical wiregrid models," IEICE Trans. Commun., vol. E93-B, no.10, pp.2611-2618, Oct. 2010.
- [20] K. Konno, Q. Chen, K. Sawaya, and T. Sezai, "Optimization of block size for CBFM in MoM," IEEE Trans. Antennas Propag., vol.60, no.10, pp.4719-4724, Oct. 2012.
- [21] K. Konno, H. Katsuda, K. Yokokawa, Q. Chen, K. Sawaya, and Q. Yuan, "Quantitative study of computing time of direct/iterative solver for MoM by GPU computing," IEICE Commun. Express, vol. 2, no. 8, pp. 359-364, 2013.
- [22] K. Konno, Q. Chen and R.J. Burkholder, "Numerical Analysis of Large-Scale Finite Periodic Arrays Using a Macro Block-Characteristic Basis Function Method," IEEE Trans. Antennas Propag., vol .65, no. 10, Oct. pp.5348-5355, 2017.
- [23] M. G. Floquet, ``Sur les equations differentielles lineaires a coefficients periodiques," Annale Ecole Normale Siiperieur, pp. 47-88, 1883.
- [24] T. F. Eibert, J. L. Volakis, D. R. Wilton and D. R. Jackson, "Hybrid FE/BI modeling of 3-D doubly periodic structures utilizing triangular prismatic elements and an MPIE formulation accelerated by the Ewald transformation," IEEE Trans. Antennas Propag., vol. 47, no. 5, pp. 843-850, May 1999.
- [25] I. Stevanoviae and J. R. Mosig, "Periodic Green's function for skewed 3-D lattices using the Ewald transformation," Microw. Opt. Tech. Lett., vol. 49, no. 6, pp. 1353-1357, Jun. 2007.
- [26] J. Su, X.-W. Xu, M. He, and K. Zhang, "Integral-equation analysis of frequency selective surfaces using Ewald transformation and lattice symmetry" Prog. Electromagn. Res., vol. 121, pp. 249-269, 2011.
- [27] D. Sievenpiper, L. Zhang, R. F. J. Broas, N. G. Alexopolous, and E. Yablonovitch, ``High-impedance electromagnetic surfaces with a forbidden frequency band," IEEE Trans. Microw. Theory Tech., vol. 47, no. 11, pp. 2059-2074, Nov. 1999.
- [28] K. M. Shum, Q. Xue, C. H. Chan, and K. M. Luk, "Investigation of microstrip reflectarray using a photonic bandgap structure," Microw. Optical Technol. Lett., vol. 28, no. 2, pp. 114-116, Jan. 2001.
- [29] K. Konno, N. Haga, J. Chararothai, Q. Chen, N. Nakamoto, and T. Takahashi, "A Novel Method of Moments for Numerical Analysis of Antennas Over 2-D Infinite Periodic Array of Scatterers," IEEE Trans. Antennas Propag., vol. 72, no. 1, pp. 50-60, 2024.
- [30] W. C. Chew, Waves and Fields in Inhomogeneous Media, IEEE Press, NY 1995.
- [31] W. C. Chew, J. L. Xiong, and M. A. Saville, ``A matrix-friendly formulation of layered medium Green's function," IEEE Antennas Wireless Propag. Lett., vol. 5, pp. 490-494, 2006.
- [32] Y. P. Chen, J. L. Xiong and W. C. Chew, ``A mixed-form thin-stratified medium fast-multipole algorithm for both low and mid-frequency problems," IEEE Trans. Antennas Propag., vol. 59, no. 6, pp. 2341-2349, June 2011.
- [33] Y. P. Chen, W. C. Chew, and L. Jiang, ``A new Green's function formulation for modeling homogeneous objects in layered medium," IEEE Trans. Antennas Propag., vol. 60, no. 10, pp. 4766-4776, Oct. 2012.