# Graph Coloring-based Pilot Assignment for User-cluster-centric Cell-free Massive MIMO Systems

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***Abstract*— A user-cluster-centric (UCC)-based cell-free massive multiple-input multiple-output (CF-mMIMO) system, which realizes multi-user multiplexing in each user-cluster formed by neighboring users and served by surrounding antennas, guarantees system scalability and improves the user capacity. However, accurate channel estimation is necessary for multi-user multiplexing. In this paper, we propose a graph coloring (GC)-based pilot assignment method to efficiently reuse the limited number of orthogonal pilots. It is verified by simulation that our proposed GC-based pilot assignment can effectively avoid pilot contamination.**

***Keywords—Cell-free, user-cluster, graph coloring, pilot assignment***

## I. INTRODUCTION

Cell-free massive multiple-input multiple-output (CF-mMIMO) system with a large number of distributed antennas (hereafter simply called "antennas") uniformly serving a relatively small number of users has attracted great attention [1], [2]. However, the scalability in terms of fronthaul capacity and the number of users that can be served is the important issue of CF-mMIMO. To achieve the system scalability, user-centric (UC) [3] and user-cluster-centric (UCC) [4]-based CF-mMIMO are proposed. The former can be seen as a special single-user-cluster form for the latter. In UCC-based CF-mMIMO, interference suppressing (IS) multi-user multiplexing, e.g. interference suppressing multi-user zero-forcing (IS-MU-ZF) [4], is considered to eliminate the interference not only inside the cluster but also between clusters.

In order to perform IS-MU-ZF well to achieve a higher user capacity, accurate channel information is necessary for the weight calculation of IS-MU-ZF. However, due to the limited number of orthogonal pilot sequences in the channel estimation phase of the CF-mMIMO system, pilot contamination caused by the reuse of pilots degrades the estimation performance. To mitigate this problem, in this paper, we propose a graph coloring [5] (GC)-based pilot assignment method for the UCC-based CF-mMIMO which is designed to assign orthogonal pilot sequences as much as possible to clusters with strong mutual interference. Although some previous works [6], [7] also utilize the GC algorithm for pilot assignment in the CF-mMIMO system, their assignment objects are users unlike ours (i.e., clusters).

The rest of this paper is organized as follows. In Section II, we introduce the system model of UCC-based CF-mMIMO. In Section III, we describe the proposed GC-based pilot assignment method. In Section IV, the computer simulation results show the validity of the proposed GC-based pilot assignment method. In Section V, we give some conclusions and future works.

## II. SYSTEM MODEL

For simplicity, in this paper, we consider a UCC-based CF-mMIMO system in a 1×1 square-shaped service area (which can be expanded to any physical scale). In the system, $U$ users and $A(\geq U)$ antennas are randomly located, and $K$ user-clusters are formed to implement IS-MU-ZF in each user-cluster simultaneously. The channel between each user and antenna is characterized as distance-dependent path loss, lognormal shadowing and Rayleigh fading. Assuming time division duplex mode, the channels of uplink and downlink are symmetrical. In this paper, we only discuss uplink-based channel estimation and evaluate the capacity performance of the uplink.

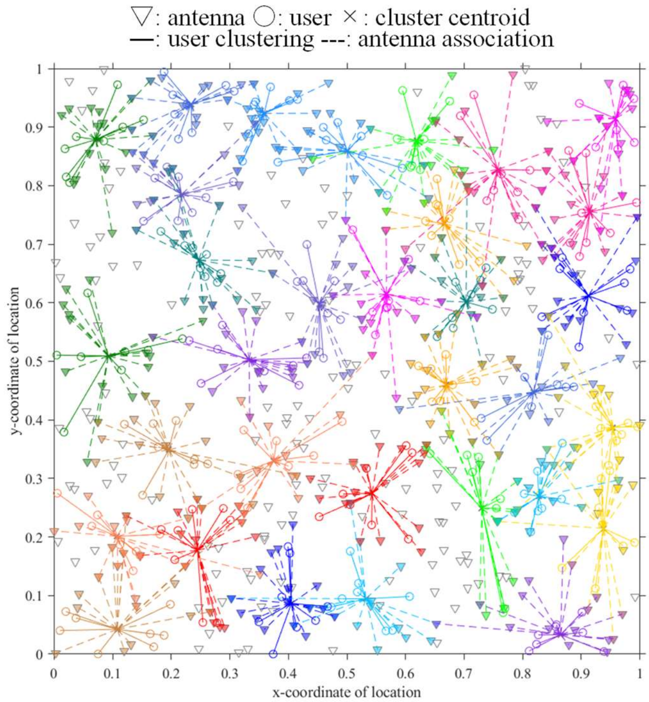


Fig. 1. An example of the constructed user-clusters with antenna association ($A$=512, $U$=256, $U_k$=8, $A_k$=16).

### *A. User-cluster Construction*

We group neighboring users with potentially strong mutual interference into clusters based on their locations. Note that the user location information is assumed to be available at the base station. This is due to the fact that the vast majority

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of user devices now have integrated global positioning system (GPS) functionality and can therefore share their location information with the base station. Considering the scalability issue, to reduce the total computational complexity required for IS-MU-ZF in each user-cluster, we utilize the constrained K-means algorithm [8] to limit the maximum number $U_k$ of users in any cluster $k$. Accordingly, the user-cluster sizes can be similar or even uniform. Then, $A_k(\geq U_k)$ antennas are associated with each user-cluster. Specifically, in each user-cluster, each user is competitively associated with $\lfloor A_k/U_k \rfloor$ antennas based on the lowest large-scale channel loss (i.e., path loss and shadowing). Note that, considering the energy consumption to activate the antennas, we allow antennas with low large-scale channel loss to belong to different clusters simultaneously. For simplicity, in this paper, we assume that $U$ and $A_k$ are the integer multiples of $U_k$. An example of the constructed user-clusters with antenna association is shown in Fig. 1.

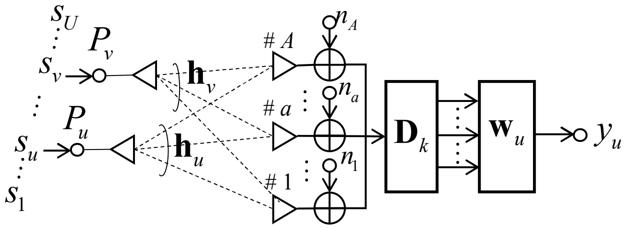



Fig. 2. Uplink transmission using partial IS-MU-ZF [4].

### B. *Partial IS-MU-ZF-based Uplink Transmission*

For uplink transmission, the partial IS-MU-ZF [4] is utilized to eliminate the interferences inside and outside each user-cluster, and its transmission model for user $u$ in user-cluster $k$ is shown in Fig. 2. The uplink received signal after postcoding is expressed for user $u$ as

$$y_u = \mathbf{w}_u \mathbf{D}_k \mathbf{h}_u \sqrt{P} s_u + \sum_{v=1, v\neq u}^{U} \mathbf{w}_u \mathbf{D}_k \mathbf{h}_v \sqrt{P} s_v + \mathbf{w}_u \mathbf{D}_k \mathbf{n}\,, \quad (1)$$

where $\mathbf{w}_u \in \mathbb{C}^{1\times A}$, $\mathbf{h}_u = [h_{1,u}, \ldots, h_{a,u}, \ldots, h_{A,u}]^T \in \mathbb{C}^{A\times 1}$, and $\mathbf{n}$ are respectively the partial IS-MU-ZF weight vector of user $u$, the channel gain vector between user $u$ and all antennas, and the noise vector with each element ($n_a$) being characterized by zero-mean complex Gaussian random variable with variance $2\sigma^2$. $P_u$ and $s_u$ are the transmit power and the transmit data symbol with unit variance for user $u$, respectively. $\mathbf{D}_k$=diag$(d_1,\ldots,d_a,\ldots,d_A)$ is the antenna association matrix for cluster $k$, where $d_a$=1 or 0 represents whether antenna $a$ belongs to cluster $k$ or not, respectively.

The uplink partial IS-MU-ZF weight vector for user $u$ in user-cluster $k$ can be derived as

$$\mathbf{w}_u = P_u \mathbf{h}_u^H \mathbf{D}_k \left( \sum_{v\in(\mathcal{S}_k \cup \mathcal{N}_k)} P_v \mathbf{D}_k \mathbf{h}_v \mathbf{h}_v^H \mathbf{D}_k \right)^{\dagger}, \quad (2)$$

where $\mathcal{S}_k$ is the set of users in user-cluster $k$ and $\mathcal{N}_k$ is the set of interfering users for user-cluster $k$ and is a target of interference suppression. In this paper, each user in $\mathcal{N}_k$ is determined by an adaptive interfering user selection method proposed in our previous work [4]. The user capacity of user $u$ can be computed by

$$C_u = \log_2 \left( 1 + \frac{P_u \left| \mathbf{w}_u \mathbf{D}_k \mathbf{h}_u \right|^2}{\sum_{v=1, v\neq u}^{U} P_v \left| \mathbf{w}_u \mathbf{D}_k \mathbf{h}_v \right|^2 + \left\| \mathbf{w}_u \right\|_{\mathrm{F}}^2} \right). \quad (3)$$

## III. GC-based Pilot Assignment Method

### A. *Problem Statement*

Assuming that a limited number $\tau$ ($U\geq\tau\geq U_k$) of mutually orthogonal pilot sequences with length $\tau$ are used for the well-known least-square channel estimation [9], the estimated channel gain between antenna $a$ and user $u$ is

$$\hat{h}_{a,u} = h_{a,u} + \sum_{v\in\mathcal{P}_u, v\neq u} h_{a,v} + \frac{1}{\sqrt{P_u}\,\tau} \boldsymbol{\phi}_u^H \mathbf{n}_a^p\,, \quad (4)$$

where $\boldsymbol{\phi}_u \in \mathbb{C}^{\tau\times 1}$ is the pilot sequence of user $u$ with $\|\boldsymbol{\phi}_u\|^2 = \tau$, $\mathbf{n}_a^p \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}_\tau, \sigma^2 \mathbf{I}_\tau)$ is the noise vector, $\boldsymbol{\phi}_u^H \mathbf{n}_a^p \sim \mathcal{N}_{\mathbb{C}}(\mathbf{0}_\tau, \sigma^2 \tau \mathbf{I}_\tau)$, and $\mathcal{P}_u$ is a set of users who are assigned with same pilot as user $u$. The first term and the second term in (4) represent the actual channel gain and the error caused by so-called pilot contamination, respectively.

The partial IS-MU-ZF weight vector $\mathbf{w}_u$ in (2) is computed by replacing $\mathbf{h}_u$ by estimated channel gain vector $\hat{\mathbf{h}}_u = [\hat{h}_{1,u}, \ldots, \hat{h}_{a,u}, \ldots, \hat{h}_{A,u}]^T$. The presence of channel estimation error increases the inter-user interference as well as the noise enhancement, both due to the pilot contamination. If $\tau$ is sufficiently large, the pilot contamination can be avoided. However, this costs a significant overhead due to the long pilot sequence transmission. Therefore, it is necessary to apply a pilot assignment algorithm which can effectively minimize the second term in (4) under the constraint on $\tau$.

### B. *Proposed GC-based Pilot Assignment Method*

We apply the well-known graph coloring (GC) algorithm [6] to solve the above mentioned problem and propose a GC-based pilot assignment method. In this proposed pilot assignment method, the available orthogonal pilots are divided into different exclusive pilot groups (represented by different colors in GC algorithm). In order to reduce the impact of channel estimation error on partial IS-MU-ZF weight, we consider assigning different pilot groups (colors) as much as possible to adjacent clusters that may interfere strongly with each other.

Specifically, we start by constructing the undirected graph consisting of vertices and edges, where the vertices will be colored, based on the constructed user-clusters (with antenna association). First, the cluster centroid (the mean of users' locations in the user-cluster) is considered as the vertex for the undirected graph. Then, from [4], users in surrounding user-clusters sharing the same antenna(s) as the user-cluster of interest are treated as interfering users to be suppressed by partial IS-MU-ZF in (3). Therefore, if these user-clusters sharing the antenna(s) are assigned the same pilot, the channel estimation error caused by pilot contamination will seriously affect the interference suppression effect of IS-MU-ZF. To avoid this, two vertices which are connected by an edge should be assigned with different colors. For instance, in Fig. 3(a), we show an undirected graph for the constructed user-clusters shown in Fig. 1.

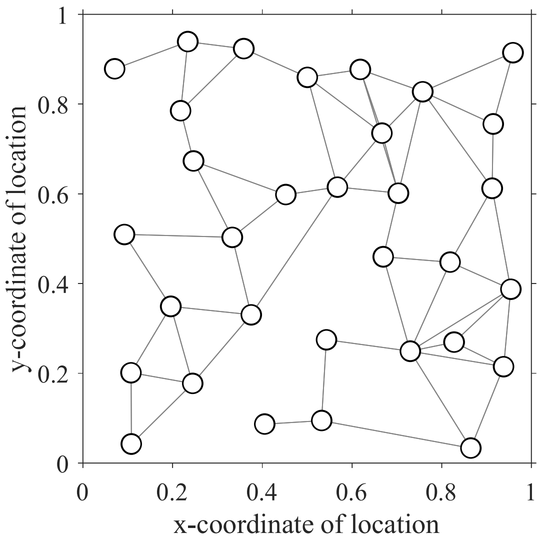
Furthermore, the limited number $N$=ceil($\tau/U_k$) of colors to be assigned is determined in advance. Then, $N$ colors are assigned based on a greedy method with following steps.

(1) Assign the current least used color to the uncolored vertex with the highest number of edges.

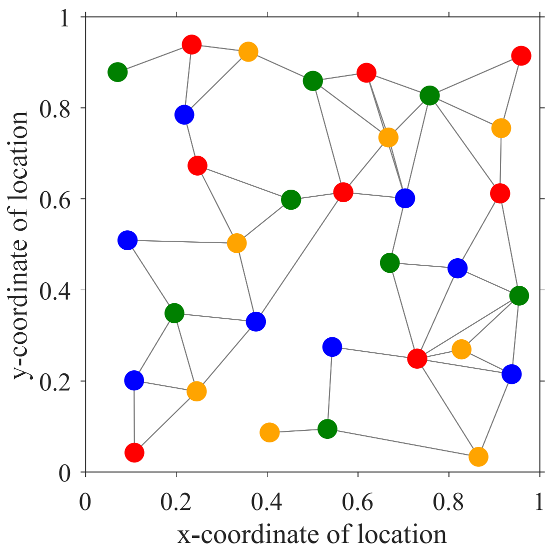
(2) Assign the current least used color to the uncolored vertex with the highest number of edges connected with the previously colored vertex.

(3) If no vertex can be assigned a color according to (2), then go back to (1). If no vertex can be assigned a color according to (1), then end.

The coloring result of undirected graph shown in Fig. 3(a) is illustrated in Fig. 3(b). It can be seen that, for the case of $K$(=256/8)=32 and $N$(=32/8)=4 case, vertices connected by edges are assigned different colors, and the number of times that each color has been assigned is the same.




(a) undirected graph




(b) coloring result

Fig. 3. Result of the proposed GC-based pilot assignment for the constructed user-clusters shown in Fig. 1 ($K$=32, $\tau$=32).

## IV. Simulation Rresults

In this section, the computer simulation is carried out to confirm the effectiveness of our proposed GC-based pilot assignment method. The simulation settings are shown in Table I. In detail, a randomly generated antenna location pattern is fixed for all the simulation trials. For each value of $U$, the user location pattern is randomly generated 100 times, and shadowing loss and Rayleigh fading are also randomly generated for each user location pattern. User-clusters are also constructed once for each user location pattern. In addition, different values of $\tau$ are considered for each user location pattern to calculate the user capacities using (3). Note that, in this paper, the overhead of pilots is not considered in the capacity calculation of simulation. That is for any value of $\tau$, the capacity is always calculated according to (3), only reflecting the influence of the channel estimation accuracy. The transmit power $P_u$ for each user is represented by the normalized transmit signal-to-noise ratio (SNR), which is defined as the received SNR when the transmitter-receiver distance is equal to the side length of the communication area of 1×1.

Table. 1 Simulation settings

| Parameter | Value |
|---|---|
| Communication area size | Normalized 1×1 |
| Antenna and user distribution | Random (uniform distribution) |
| No. of antennas ($A$) | 512 |
| No. of users ($U$) | 32, 64, 128, 192, 256, 320, 384, 448, 512 |
| No. of user location generation for each $U$ | 100 |
| No. of users in each cluster ($U_k$) | 8 |
| No. of antennas in each cluster ($A_k$) | 16 |
| Path loss exponent | 3.5 |
| Standard deviation of shadowing loss | 8 [dB] |
| Normalized transmit SNR ($P_u$) | -30 [dB] |
| No. of orthogonal pilot sequences ($\tau$) | 8, 16, 32, 64, 512 |

In Fig. 4, the cumulative distributed function (CDF) of uplink user capacity achieved by using our proposed GC-based pilot assignment is plotted with $\tau$ as a parameter for the case of $U$=256. For comparison, CDF of user capacity with a random assignment method (which randomly assigns the colors to each vertex of the same undirected graph as constructed in our proposed method) is also plotted. As can be seen from Fig. 4 that, as the value of $\tau$ becomes larger, the achievable uplink user capacity increases for both assignment methods. Note that, for the case of $\tau$=$U_k$=8, all 8 pilots are reused in each cluster, and for the case of $\tau$=512, orthogonal pilots can be assigned to every user. Thus, the results of the two pilot assignment methods are the same for these two cases. Accordingly, for simplicity, in Fig. 4, we only plot one CDF curve for both $\tau$=8 and 512. In addition, for the cases of $\tau$=16, 32, and 64, it can be clearly seen that our proposed GC-based pilot assignment method always outperforms the random assignment method, and that it achieves a very close result to the ideal case ($\tau$=512) while using a smaller number of pilots, i.e., $\tau$=32 pilots.

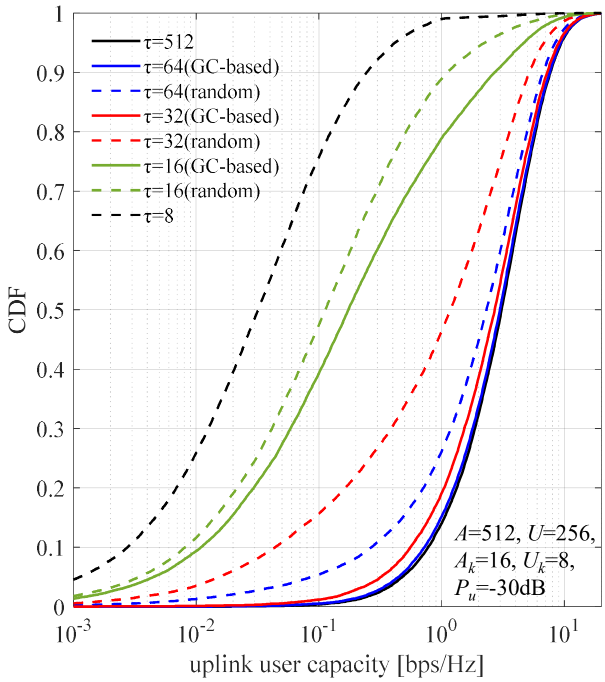



Fig. 4. CDF of uplink user capacity achievable with our proposed GC-based pilot assignment and the random assignment ($U$=256).

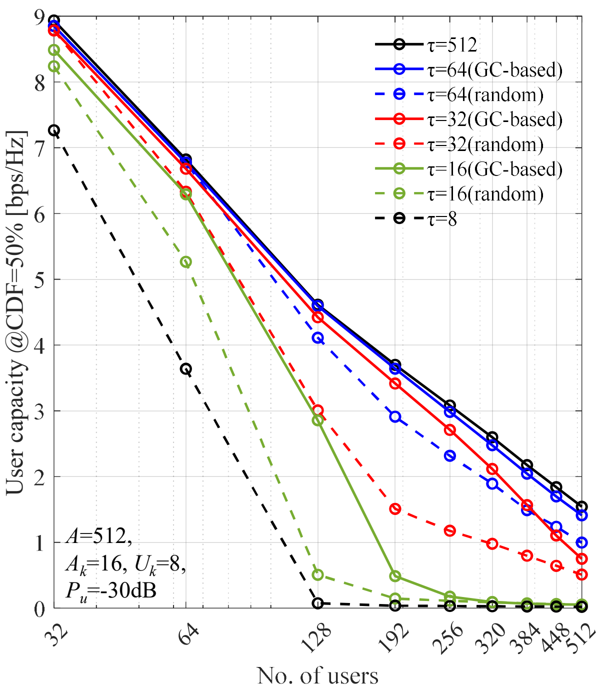



Fig. 5. Uplink user capacity achievable with our proposed GC-based pilot assignment and the random assignment at CDF=50% as a function of $U$.

To further verify the effectiveness of our proposed GC-based pilot assignment method, we plot the uplink user capacity at CDF=50% as a function of $U$ in Fig. 5. It can be seen from Fig. 5 that as $U$ increases, the user capacity decreases. This is because first, the severer interference among clusters is produced due to the increased number of clusters and the reduced cluster spacing, and second, the severer pilot contamination due to the increased number of times reusing of the same pilot. It can be also seen from Fig. 5 that regardless of $U$, our proposed GC-based pilot assignment method achieves a higher user capacity than the random method, and approaches the user capacity achievable when $\tau$=512 by using $\tau$=32 pilots, as we have discussed referring to Fig. 4.

## V. Conclusions

In this paper, we proposed a GC-based pilot assignment method to reuse a limited number of orthogonal pilots to improve the channel estimation for the UCC-based CF-mMIMO system. It was confirmed by computer simulation that, our proposed GC-based method achieves a higher user capacity compared with the random assignment method and can well approach the capacity achievable with a sufficient number of pilots (i.e., $\tau$=512) by using a fairly small number $\tau$=32 of pilots.

Besides the GC algorithm, many other algorithms can be used for the pilot assignment. Improving pilot allocation method is an interesting topic for our future study. In addition, since the impact of pilot overhead on capacity was not discussed in this paper, this will also be investigated in our future study.

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