Accurate DOA Estimation Using Array Antenna With Arbitrary Geometry

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Abstract—The so called universal steering vector (USV) whose locus is equivalent to the array element pattern is applied to the direction of arrival (DOA) estimation by multiple signal classification (MUSIC) algorithm. It is shown that if the USV which includes the effect of the mutual coupling between the array elements is used, the compensation for the received voltage to remove the effect of the mutual coupling is not required any more. The USV for array antennas with arbitrary geometry is derived and evaluated efficiently by using the method of moments (MoM) so that the DOA estimation can be performed accurately by using the array antenna with arbitrary geometry. Numerical examples of the DOA estimation by a dipole array antenna, and an antenna array composed of a monopole antenna and a planar inverted-F antenna (PIFA) mounted on a mobile handset are presented to demonstrate the effectiveness of the proposed method.

Index Terms—Antenna array mutual coupling, antenna arrays, array signal processing, direction of arrival (DOA) estimation, method of moments (MoM), mobile antennas.

I. INTRODUCTION

IT HAS BEEN pointed out that the accuracy of the direction of arrival (DOA) estimation using the multiple signal classification (MUSIC) algorithm is degraded due to the effect of the mutual coupling between the array elements. Therefore, many efforts have been made to remove the effect of the mutual coupling from the received voltage at the array terminals or from the steering vector, and then to apply the compensated received voltage or the compensated steering vector to evaluate the MUSIC spectrum [1]–[5].

The open-circuit voltage method has been proposed to compensate the effect of the mutual coupling by multiplying the received voltage vector by the impedance matrix of the array elements including the load impedance [6]. However, it has been pointed out that this compensation method is effective in the case of small dipole antenna but is not valid generally because the scattering effect of the antenna elements in the open-circuit state is ignored [2], [7], [8]. The mutual impedance has been redefined by taking the open-circuit scattering effect into account [2], [7], but the current distribution has to be estimated. The estimation becomes difficult when the structure of the array elements is complicated because the current distribution on each

antenna element depends on the direction and the polarization of the incident wave, as well as the geometry of the antenna elements. The compensation for the received voltage has been also carried out by the MoM [8], where the DOA of the incident waves is required. Generally speaking, it is difficult to compensate the received voltage to reconstruct the signal subspace because the compensation requires the information about the DOA of the incident wave in prior which is unavailable in DOA estimation.

Approaches of using the actual array manifold defined as the array response have been suggested to find the DOA [9]. The array manifold is equivalent to the array element pattern in our opinion, and the method to obtain the array manifold is also called as the calibration method. In most cases, the actual array manifold is obtained by the measurement and requires huge storage to be saved in prior. Therefore, efforts have been made to get more exact array manifold with a less storage without considering the electromagnetic characteristics of the array antenna [9]–[12]. The method to obtain the array manifold from a fundamental electromagnetic perspective has also been proposed where the actual array manifold is calculated by the MoM to compensate the steering vector [5], and the problem of the cost of a large amount of storage is solved by using the distortion matrix with a limited size which is derived by comparing the actual array manifold (steering vector) with the ideal manifold in the limited number of directions, and the matrix is then approximately assumed to be independent of the DOA angle for saving the CPU time to calculate the matrix for every spatial spectrum.

It has been found that the steering vector which is defined to be the array element pattern is affected greatly by the mutual coupling and the steering vector has been evaluated by using the EMF method [13], and analytical formulas [14] for some simple structures of the antenna arrays, but how to obtain the steering vector for general cases has not been clarified.

In this paper, the so called universal steering vector (USV) whose locus is equivalent to the array element pattern is applied to the DOA estimation by means of MUSIC algorithm so that the received voltage at the terminals of array antennas can be used directly to calculated the MUSIC spectrum without compensation. An effective approach to calculate the USV without being saved at all searching directions is presented with the help of the full wave numerical analysis using the method of moments (MoM) for array antennas with arbitrary geometry. Also, the relationship between the USV and the conventional steering vector (CSV) is investigated. Finally, numerical examples of the DOA estimation by using a dipole array antenna, as well as an

array antenna composed of a monopole antenna and a planar inverted-F antenna (PIFA) mounted on a mobile handset are presented to demonstrate the effectiveness of the proposed method.

II. DOA ESTIMATION BY MUSIC ALGORITHM WITH USV

The received signal space of the M array elements can be divided into two subspaces. One is the incident signal subspace spanned by the L incident signal eigenvectors and the other is the noise subspace spanned by M-L noise eigenvectors.

Both the signal eigenvectors and the noise eigenvectors can be calculated by the covariance matrix of the received voltage $[V^r]$ at the terminals of the antenna elements as

$$[R_{vv}] = E\left([V^r][V^r]^H\right) \tag{1}$$

where $E()$ denotes the statistical expectation and superscript H denotes the complex conjugate transpose.

Conventionally, the orthogonal property between the signal eigenvectors and the noise eigenvectors is utilized to estimate the DOA by searching the peaks of the MUSIC spectrum given by

$$P_{\text{MU}}(\theta, \phi) = \frac{[A(\theta, \phi)]^H [A(\theta, \phi)]}{[A(\theta, \phi)]^H [E_N][E_N]^H [A(\theta, \phi)]} \tag{2}$$

where θ and ϕ and are the searching angles, and the polarization of the incident wave is assumed to be known. $[E_N]$ is the $M \times (M - L)$ matrix whose columns are the M noise eigenvectors of the covariance matrix $[R_{vv}]$, and $[A]$ represents the CSV.

Most of the previous researches have tried to convert the received voltage vector $[V^r]$ into the incident voltage vector $[V^{\text{inc}}]$, where $[V^{\text{inc}}]$ does not include the mutual coupling effect and has size of M only representing the voltage at each antenna element terminal. Then $[V^{\text{inc}}]$ is used to calculate $[R_{vv}]$ in (1) and the MUSIC spectrum in (2). However, the transformation from $[V^r]$ into $[V^{\text{inc}}]$ is possible only for special cases such as the dipole array antenna where the compensation matrix is independent of the DOA. The compensation for mutual coupling effect usually requires the incident wave information such as the DOA and the polarization, but this information is practically not available. In this work, the USV defined by (9) is used to estimate the DOA without compensating the received voltage and the MUSIC spectrum is calculated by using the following:

$$P_{\text{MU}}(\theta, \phi) = \frac{[A^u(\theta, \phi)]^H [A^u(\theta, \phi)]}{[A^u(\theta, \phi)]^H [E_N][E_N]^H [A^u(\theta, \phi)]}. \tag{3}$$

Since the USV can be evaluated for array antennas with arbitrary geometry as shown in Section III, the DOA estimation by means of the MUSIC algorithm can be used for any type of receiving array antennas including the effect of the mutual coupling between the array elements. It should be noted that the steering vector also depends on the polarization of the incident wave but the polarization of the incident wave is assumed to be known in this paper.

III. FORMULATION OF UNIVERSAL STEERING VECTOR

A. CSV

Assuming that an M-element linear antenna array is used as the receiving antenna to estimate the DOA of incident waves from L different directions. The received voltage vector with dimension of M can be expressed by

$$[V] = \sum_{j=1}^{L} [A(\theta_j, \phi_j)] f(\theta_j, \phi_j) s(\theta_j, \phi_j) + [V^n] \tag{4}$$

where $f(\theta_j, \phi_j)$ represents the ideal isolated element pattern which is the same for all elements in array antenna having the same elements. $s(\theta_j, \phi_j)$ denotes the electric field of jth incident wave at the origin and $[V^n]$ is the voltage vector with dimension M caused by the white noise. $[A(\theta_j, \phi_j)]$ is the CSV with dimension M and the jth element of $[A(\theta_j, \phi_j)]$ is given by

$$a_i(\theta_j, \phi_j) = e^{j\vec{k}_j \cdot (\vec{r}_i - \vec{r}_0)} \tag{5}$$

where $\vec{r}_i$ is the position vector of ith array element, $\vec{r}_0$ is the reference location for the phase of the incident wave and $\vec{k}_j$ is the wave number vector of the jth incident wave from direction (θ_j, ϕ_j). The CSV, which depends on the position of the array elements and the direction of the incident wave, does not include the mutual coupling between the array elements.

B. USV

The ith element of the USV is denoted by $a_i^u(\theta, \phi)$ which is the complex received voltage at the terminal of the ith array element due to the incident wave from direction (θ, ϕ) and is equivalent to the element pattern of the ith array element. Although the USV can be obtained by the measurement of the actual complex array element pattern of each element directly, the accurate measurement of the array element pattern is not easy [15] and requires the huge storage to save the measured data. An efficient method to evaluate the USV by using method of moments (MoM) is presented in the following.

The array antenna with arbitrary geometry is shown in Fig. 1 as an example of the receiving array for the DOA estimation. In the MoM analysis, the ith element of the array is divided into N_i^s segments and the total number of the segments of the array is $N(= \sum_{i=1}^{M} N_i^s)$. N_i^s depends on the segment size and antenna structure. The segment size less than $\lambda/(2\pi)$ for wire structure in Richmond's MoM procedure is necessary to obtain a satisfactory accuracy. Then, the matrix equation for unknown currents on all segments is obtained as

$$[Z][I] = [V^{\text{inc}}(\theta, \phi)] \tag{6}$$

where $[I]$ is the current vector with dimension of N representing the unkown currents on all segments. $[Z]$ is the $N \times N$ impedance matrix whose element z_{ij} represents the mutual impedance between ith and jth segments. The impedance matrix $[Z]$ is independent of the incident wave which is evaluated
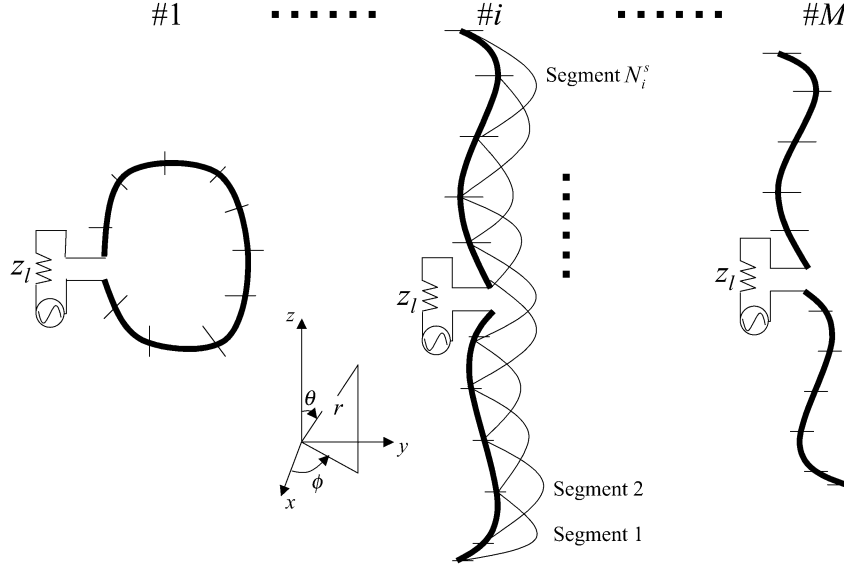
Fig. 1.   Array antenna with arbitrary geometry for DOA estimation.

numerically by using the full wave electromagnetic analysis. $[V^{\text{inc}}(\theta, \phi)]$ is the N voltage vector representing the inner product of the weighting functions and the incident electric field from direction (θ, ϕ) for all segments. The Richmond's method where the piece-wise sinusoidal is used as basis and weighting functions is employed in this work, the detail for evaluations of $[Z]$ is given in [17], and the evaluation of $[V^{\text{inc}}(\theta, \phi)]$ of ith element is simple analytic form which is omitted here. Once $[V^{\text{inc}}(\theta, \phi)]$ vector and $[Z]$ matrix are evaluated, the unknown current vector $[I]$ can be obtained by

$$[I] = [Y][V^{\text{inc}}(\theta, \phi)] \tag{7}$$

where $[Y]$ is the inverse matrix of $[Z]$, i.e., the admittance matrix. Since the currents at the terminal segments of the antenna element are part of $[I]$, they can be extracted from (7) and expressed as

$$[I^{\text{ter}}] = [Y^{\text{ter}}][V^{\text{inc}}(\theta, \phi)] \tag{8}$$

where $[I^{\text{ter}}]$ is the current vector with dimension of M representing the currents at the terminal segments and $[Y^{\text{ter}}]$ with dimension of $M \times N$ is the part of $[Y]$ corresponding to the mutual admittances between the segments on the terminals and all the segments of the array elements. It is clear that $[Y^{\text{ter}}]$ includes the effect of the mutual coupling not only from the terminal segments, i.e., the feed points, but also from all the segments of the antennas.

Assuming that the terminal of the array element is loaded by an impedance of z_l, the USV which represents the received voltage taking account of the effect of the mutual coupling between the array elements is given by

$$[A^u(\theta, \phi)] = z_l[Y^{\text{ter}}][V^{\text{inc}}(\theta, \phi)]. \tag{9}$$

Although the $[A^u(\theta, \phi)]$ in (9) has to be evaluated for every searching angle (θ, ϕ) in the spatial spectrum, since $[Y^{\text{ter}}]$ is no relation with (θ, ϕ) and can be calculated and stored before the DOA estimation, the calculation of $[A^u(\theta, \phi)]$ in (9) only

requires evaluation of $[V^{\text{inc}}(\theta, \phi)]$ and a multiplication of $M \times N$ matrix by N vector, whose computational cost is not too large to be calculated.

C. Relationship Between USV and CSV

It is noted that the CSV is equal to the incident voltages at the terminal segments of array antenna and is the part of the $[V^{\text{inc}}(\theta, \phi)]$. If each array element is not divided, i.e., $N_i^s = 1$, the CSV is equal to $[V^{\text{inc}}(\theta, \phi)]$. However, in order to calculate the mutual coupling accurately by using MoM, the array antenna is usually divided into $N_i^s(> 1)$ segments to meet the requirement that the dipole segment size should be less than $\lambda/(2\pi)$. The relationship between the CSV and $[V^{\text{inc}}(\theta, \phi)]$ can be expressed as the follows:

$$[V^{\text{inc}}(\theta, \phi)] = [T(\theta, \phi)][A^c(\theta, \phi)] \tag{10}$$

where $[A^c(\theta, \phi)]$ represents the CSV with dimension of M, and $[T(\theta, \phi)]$ is the transformation matrix with dimension of $N \times M$. Obviously, $[T(\theta, \phi)]$ depends on the searching direction (θ, ϕ). Substituting (10) into (9), the following expression can be obtained:

$$[A^u(\theta, \phi)] = z_l[Y^{\text{ter}}][T(\theta, \phi)][A^c(\theta, \phi)] \tag{11}$$

and the USV can be expressed by

$$[A^u(\theta, \phi)] = [C(\theta, \phi)]^{-1}[A^c(\theta, \phi)] \tag{12}$$

where $[C(\theta, \phi)]$ defined by

$$[C(\theta, \phi)]^{-1} = z_l[Y^{\text{ter}}][T(\theta, \phi)] \tag{13}$$

is the $M \times M$ matrix and plays a role to compensate the CSV in (12). $[T(\theta, \phi)]$ in (10) and $[C(\theta, \phi)]$ in (12) are used just for explaining the relationship between the USV and the CSV as shown in (12), but not for evaluating the USV from the CSV in practice. Although the purpose of $[C(\theta, \phi)]$ in (12) is the same to the impedance matrix in the open-circuit voltage method [6] to compensate the mutual coupling either for restoring the
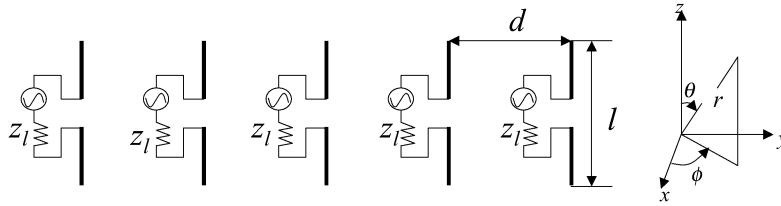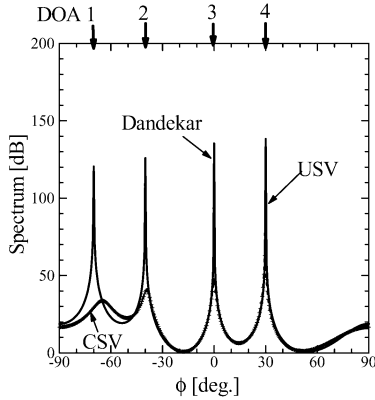
Fig. 2. Six-element dipole array.

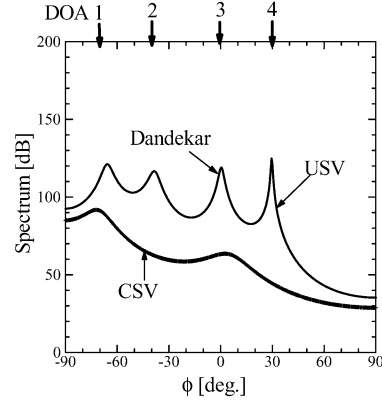Fig. 3. MUSIC spectrum by using six-element dipole array with array spacing of 0.5λ.

Fig. 4. MUSIC spectrum by using six-element dipole array with array spacing of 0.1λ.

steering vector or for restoring received signals, there is an essential different between $[C(\theta, \phi)]$ and the impedance matrix. $[C(\theta, \phi)]$ depends on each searching angle and polarization, while the impedance matrix does not. It is the difference that makes the USV compensate the CVS perfectly and estimate DOA in MUSIC algorithm accurately. Using the CSV for DOA estimation requires that all the element should be the same, while if the USV is used, the receiving array antenna with arbitrary geometry and arbitrary element spacing is acceptable.

IV. SIMULATION RESULTS

A. Dipole Array

Fig. 2 shows a six-element linear dipole array antenna for the numerical simulation. The length of dipole element is $l = \lambda/2$ and each dipole antenna is loaded by 50 Ω resistance. Four incident waves with θ polarization are assumed and the DOA of each wave is $\theta = 90°$, and $\phi = -70°$, $-40°$, $0°$, and $30°$, respectively. The SNR of all the waves is 20 dB. The MUISC spectrum obtained by the CSV, the USV and Dandekar's method [5] are compared in Figs. 3 and 4 when array spacing is $d = 0.5\lambda$ and $d = 0.1\lambda$, respectively. These figures show that the DOA is accurately evaluated by using the USV and Dandekar's method for both cases. On the other hand, the MUISC spectrum of the CVS cannot distinguish the DOA of $\phi = -70°$ in the case of $d = 0.5\lambda$ and cannot estimate any DOA in the case of $d = 0.1\lambda$.

B. Monopole-PIFA Array for Mobile Handset

Fig. 5 shows another model of array antenna for the simulation composed of a monopole antenna and a planar inverted-F antenna (PIFA) antenna. The combination of such antennas has been used in the mobile handset to achieve the diversity reception. The radiation pattern of each antenna element is different
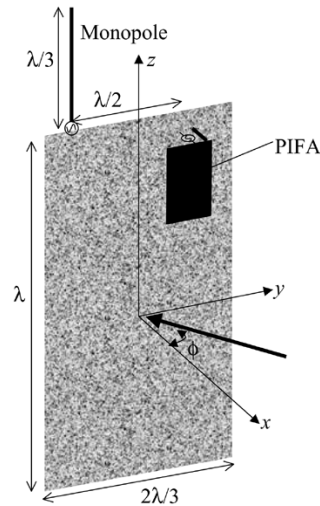

Fig. 5. Monopole-PIFA array mounted on mobile handset.

from each other and much affected by the ground plane and the mutual coupling between the elements [16]. Therefore, the DOA cannot be evaluated correctly unless the effect of ground plane and the mutual coupling is exactly included into the DOA estimation. The MUSIC spectrum for $\theta = 90°$ and $\phi = -90°$–$90°$ is evaluated for two cases. Fig. 6 shows the result of case 1 where the incident wave with vertical polarization E_θ from the direction of $\theta = 90°$ and $\phi = 70°$ is assumed, while Fig. 7 shows the result of case 2 where the incident wave with horizontal polarization E_ϕ from the direction of $\theta = 90°$ and $\phi = 0°$ is assumed. The input SNR for both cases is 20 dB. The MUISC spectrum obtained by the CSV, the USV and Dandekar's method are compared in Fig. 6 for case 1. This figure shows that the DOA is correctly evaluated by using the USV but the method
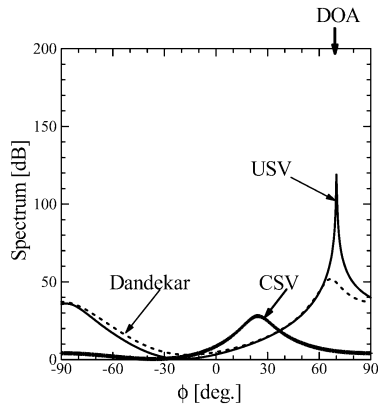
Fig. 6.　MUSIC spectrum obtained by using monopole-PIFA array. (Case 1: E_θ polarization).
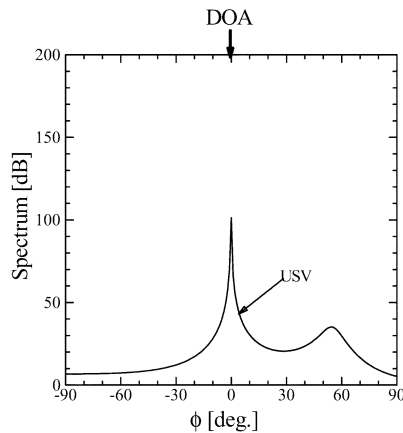

Fig. 7.　MUSIC spectrum obtained by using monopole-PIFA array. (Case 2: E_ϕ polarization).

using the CVS and Dandekar's method are invalid in this case. It indicates that the distortion matrix in [5] is effective in compensating the array antenna such as the dipole array, but cannot compensate the coupling effect in the present case where the distortion matrix strongly depends on the incident direction. Fig. 7 shows that the present method is effective where the DOA has the E_ϕ polarization. These numerical results indicate that the effect of the ground plane and the mutual coupling can be dealt with correctly by the present method.

V. CONCLUSION

The USV whose locus is equivalent to the array element pattern has been applied to the DOA estimation by means of MUSIC algorithm so that the received voltage at the terminals of the array antenna can be used directly to calculate the MUSIC spectrum. An effective approach to calculate the USV has been shown by using the method of moment for the array antenna with arbitrary geometry. The relationship between the USV and the CSV has been shown indicating that the USV can be regarded as the CSV compensated by the inverse of the

mutual coupling matrix, a dynamic matrix depending on the searching direction and polarization. Theoretically the matrix could include the mechanical and electrical errors due to the mutual coupling between the array elements. The numerical examples have been shown when a dipole array antenna and an array antenna composed of a monopole antenna and a planar inverted-F antenna (PIFA) mounted on a mobile handset are used as the receiving antenna, demonstrating the high accuracy of the present method.

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