社団法人 電子情報通信学会
THE INSTITUTE OF ELECTRONICS,
INFORMATION AND COMMUNICATION ENGINEERS

信学技報
IEICE Technical Report
A·P2007-53(2007-7)

Analysis of Dielectric Body by Using Volume Integral Equation Combined with Multi-region Iterative Method

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Abstract In this research, electromagnetic analysis of 3-D dielectric body is carried out by using the MoM. The unknown polarization current in dielectric body is expanded into rectangular blocks with overlapping volume sinusoidal functions. To accelerate the matrix-solving CPU time, the whole dielectric volume model is divided into many smaller overlapping volume sub-regions and iterative algorithm, where the overlapping volume sub-region is treated as the iteration unit, is applied to solving the matrix equation in the MoM. Some numerical results are given to show that the CPU time for solving unknown currents can be reduced effectively by multi-region iterative method.

Keyword Method of moments, Dielectric body, Multi-region iterative method, Volume integral equation

1. Introduction

Electromagnetic materials including dielectric bodies are widely involved in many problems, such as printed antenna on dielectric substrate and the power absorption by human bodies. The method of moments (MoM) is one of the effective methods for such problems. However, when the dielectric material is included, the integral equation is expressed by volume integral equation and calculating the mutual impedance between the dielectric blocks costs much of the CPU time. The point matching technique has been used to reduce the CPU time [1], but it is difficult to obtain the accurate solution of self/mutual impedance. The Galerkin-MoM analysis for dielectric scatters by using sinusoidal reaction technique was reported [2], where the sinusoidal function was used as the basis and test functions, and double volume integral was reduced to a three-dimensional integral. Recently, the double volume integral was transformed into one-dimensional integral to reduce the CPU time [3][4].

Although the CPU time of filling the self/mutual impedance can further be reduced, when the dielectric body becomes larger, the solution of matrix equation for evaluating the polarization current will cost much more CPU time than matrix-filling. Many researches have been done in order to obtain the fast current solution. The fast multipole method (FMM)[5], probably the most well known, reduces CPU time by accelerating the matrix-vector multiplication in an iterative solver. Another way to reduce CPU time is based on spatial segmentation of the original regions into smaller sub-regions. The application of the multi-region iterative method for two-dimensional or three-dimensional surface radiation and scattering problems of conducting bodies can be found in [6]-[9]. However, the application of the volume integral method combined with multi-region iterative method for three-dimensional volume radiation and scattering problems of dielectric bodies has not been reported.

In this research, electromagnetic analysis of 3-D dielectric body is carried out by using the MoM. The unknown polarization current in dielectric body is expanded into rectangular blocks with overlapping volume sinusoidal functions. To accelerate the matrix-solving CPU time, the whole dielectric volume model is divided into many smaller overlapping volume sub-regions and iterative algorithm, where the overlapping volume sub-region is treated as the iteration unit, is applied to solving the matrix equation in the MoM. It is illuminated in detail that how the multi-region iterative method is implemented on the dielectric body problem. Finally, numerical result is given to show that the CPU time for solving unknown currents can be reduced effectively by multi-region iterative method. The method's characteristics are investigated by the detailed numerical results.

2. Formulation

In the MoM, the dielectric body is divided into dielectric blocks and the mutual impedance between the blocks is expressed by a double volume integral

$$Z_{mn}^{dd} = j\omega\mu_0 \iiint_{V_m} \iiint_{V_n} \vec{g}_m(\vec{r},\vec{r}') \cdot \overline{\overline{G}}_0(\vec{r},\vec{r}') \cdot \vec{f}_n(\vec{r}) dV dV' + \delta_{m,n} \iiint_{V_m} \left[\frac{1}{\sigma + j\omega(\varepsilon_r \varepsilon_0 - \varepsilon_0)} \right] \vec{g}_m(\vec{r}) \cdot \vec{f}_n(\vec{r}) dV \quad (1)$$

where $\overline{\overline{G}}_0(\vec{r},\vec{r}')$ is the free space dyadic Green's function, and $\vec{f}_n$ and $\vec{g}_m$ denote the sinusoidal basis functions and the sinusoidal test functions for the polarization current inside the dielectric scatters, respectively. The unknown polarization current is expanded into overlapping volume sinusoidal functions in x, y and z directions.

After filling the self/mutual impedance, generally the following matrix equation can be obtained by

$$[Z][I] = [V] \quad (2)$$

By using the multi-region iterative method for solving above Eq.(2), the whole volume object is divided into smaller volume sub-regions. Considering the iterative convergence problem, the adjoining regions are overlapped generally [6]. The volume multi-region model is shown in Fig.1.

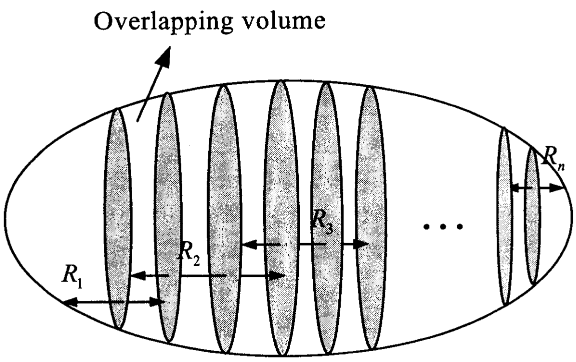

Fig.1 The multi-region model including many overlapping smaller volumes

Firstly, if the interactions from the other regions are neglected, the relationship between unknown current and excitation voltage in the first volume sub-region (R_1) can be expressed by

$$[Z_{11}][I_1] = [V_1] \quad (3)$$

where $[Z_{11}]$ denotes the self impedance in volume sub-region_1. $[I_1]$ is the unknown current coefficient vector in volume sub-region_1. If the unknown current vector $[I_1']$ in non-overlapping part of sub-region_1 is assumed to be unchanged, unknown current vector $[I_2]$ in volume sub-region_2 can be solved by

$$[Z_{22}][I_2] = [V_2] - [Z_{21}'][I_1'] \quad (4)$$

where $[Z_{21}']$ is the modified mutual impedance matrix, which represents the interaction between volume sub-region_2 (R_2) and the non-overlapping volume part of sub-region1.

Repeating the above procedures, we can obtain the iterative solution of the l^{th} volume sub-region currents by the following iterative matrix equation which can also be called forward and backward iterative method:

For the forward iterative equation:

$$\left\{ \sum_{i=1}^{l-1} [Z_{li}'][I_i']^{(m)} \right\} + [Z_{ll}][I_l]^{(m)} + \left\{ \sum_{i=l+1}^{n} [Z_{li}'][I_i']^{(m-1)} \right\} = [V_l] \quad (5)$$

For the backward iterative equation:

$$\left\{ \sum_{i=1}^{l-1} [Z_{li}'][I_i']^{(m)} \right\} + [Z_{ll}][I_l]^{(m+1)} + \left\{ \sum_{i=l+1}^{n} [Z_{li}'][I_i']^{(m+1)} \right\} = [V_l] \quad (6)$$

where l represents sub-region number. For (5), l=1,2,......., n and for (6), l.=n ,......., 2,1. m represents iterative step.

In above two equations, $[Z_{ll}]$ denotes the self impedance matrix of sub-region l (R_l). $[Z_{li}']$ denotes the modified mutual impedance matrix, which represents the interaction between sub-region l and the corresponding non-overlapping part of sub-region i. $[I_i']$ denotes the unknown current matrix in non-overlapping part of sub-region i. $[V_l]$ denotes the exciting voltage matrix in sub-region l. The superscript m denotes the m^{th} iteration. The initialized current $[I_l]^{(0)} = 0$.

The iterative error is estimated by the residual norm, which is defined by

$$\Pi_L = \|[Z][I]-[V]\| / \|[V]\| \qquad (7)$$

3. Numerical results

The model for a numerical analysis is shown in Fig.2. A wire dipole antenna is located in the vicinity of a dielectric rectangular box. The dielectric body is divided into dielectric blocks. The sinusoidal basis and test functions are also used for the dipole antenna.

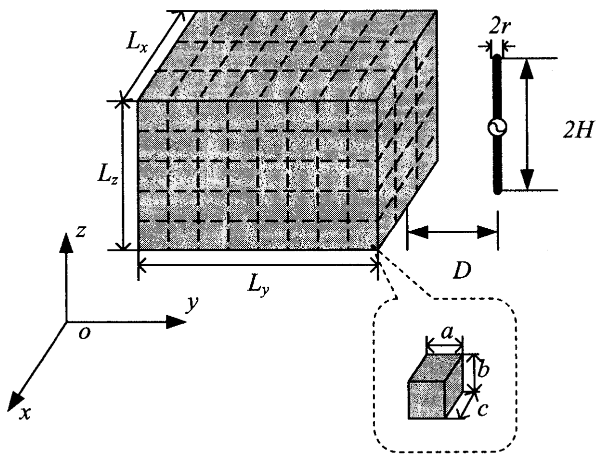

Fig.2 Simulation model of dipole antenna located in the vicinity of dielectric box

In order to demonstrate the accuracy of the volume integral method, we will use the FDTD method and the volume integral method to compute the first numerical example: L_x=10mm, L_y=10mm, L_z=120mm, D=2mm, $2H$=100mm, r=0.1mm, ε_r=16. The size of volume monopole basis function is 3.33mm×6.0mm×3.33mm. The comparison of resistance and reactance between volume integral method and FDTD method is shown in Fig.3 and Fig.4. In the volume integral method, the Gauss-Jordan method is used for solving unknown currents.

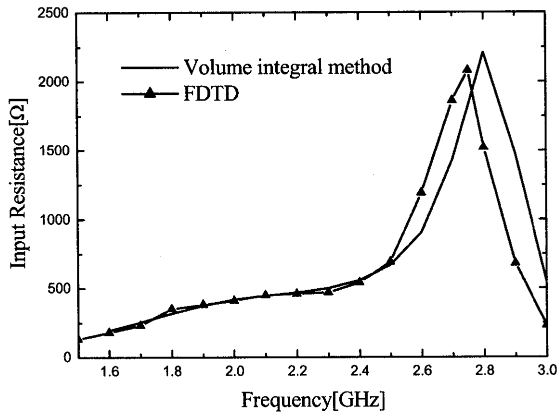

Fig.3 Resistances obtained by the volume integral method and FDTD method

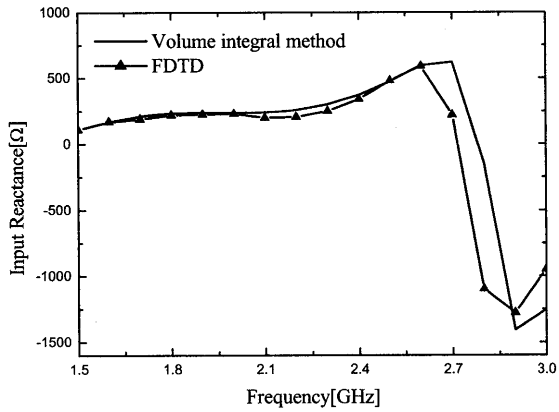

Fig.4 Reactances obtained by the volume integral method and FDTD method

From the comparison between the FDTD method and volume integral method, a good consistency can be obtained.

In order to show the validity of the multi-region iterative method on based of volume integral method, the second numerical example is given. L_x=10.5mm, L_y=112mm, L_z=10.5mm, D=1mm, $2H$=10.5mm, r=0.1mm, ε_r=5. The size of volume monopole basis function is 3.5m×3.5mm×3. 5mm. The multi-region model is shown in Fig.5. The dielectric scatter is divided into n overlapped volume sub-regions, which is shown in the following figure.

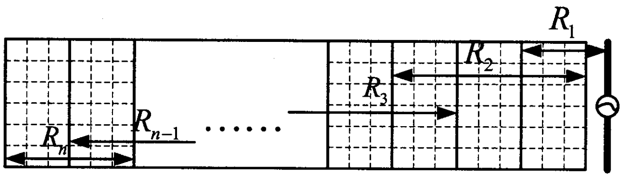

Fig.5 Multi-region model for antenna and dielectric box

Gauss-Jordan method (n=1) and multi-region iterative method (n>1) are used for the same size body. In the sub-region iterative method, the residual norm is smaller than 1×10^{-6}.

The antenna input impedance results by Gauss-Jordan method and the multi-region iterative method (n=3) are shown in Fig.8, where impedance_1 denotes the antenna input impedance without the dielectric box and impedance_2 denotes the antenna input impedance with the dielectric box.

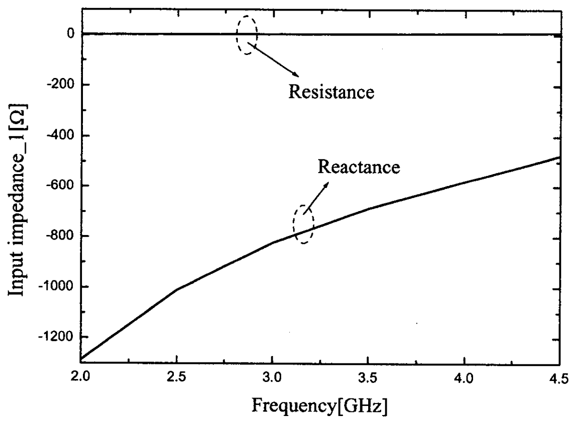

Fig.6 Antenna input impedance without the dielectric box

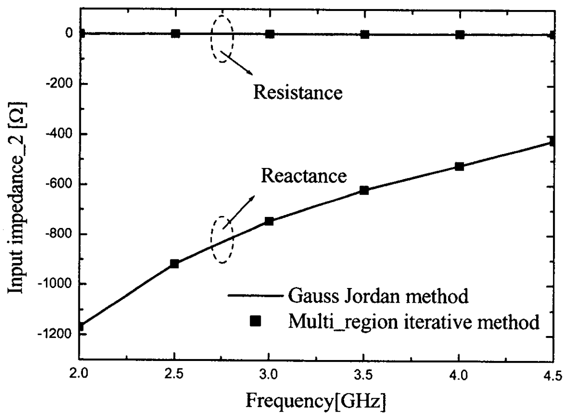

Fig.7 Antenna input impedance with the dielectric box

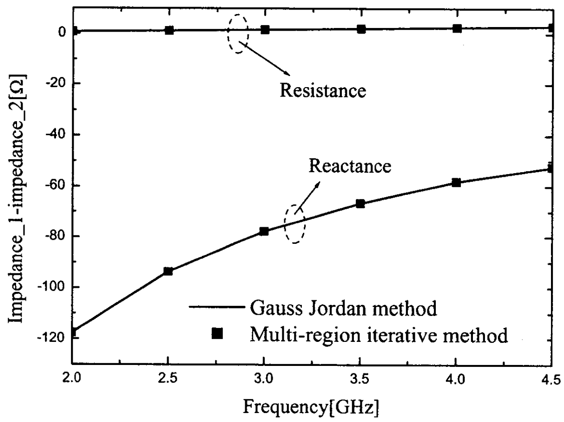

Fig.8 Input impedances obtained by Gauss-Jordan method and multi-region iterative method

From above comparison, we can see that the results from the Gauss-Jordan method and multi-region iterative method have a good agreement.

The CPU-times with different number of sub-regions when operation frequency is 4.5GHz are shown in the Tab.1 where a Pentium-III 2.8GHz PC with 1.5GB of memory is used.

Number of sub-regions	CPU time for obtaining inversing matrix [sec.]	Iterative steps	CPU time for iteration [sec.]	Whole CPU time of solving unknowns [sec.]
1	444.218			444.218
3	203.078	4	0.828	203.906
5	50.296	6	1.218	51.514
7	23.500	9	1.781	25.281
9	6.0625	10	2.203	8.265

Tab.1 CPU time results for different number of sub-regions

From above table, we can see that the whole CPU time of solving unknowns can be effectively reduced by multi-region iterative method.

The number of iteration steps versus the total number of the unknowns is shown in Fig.9.

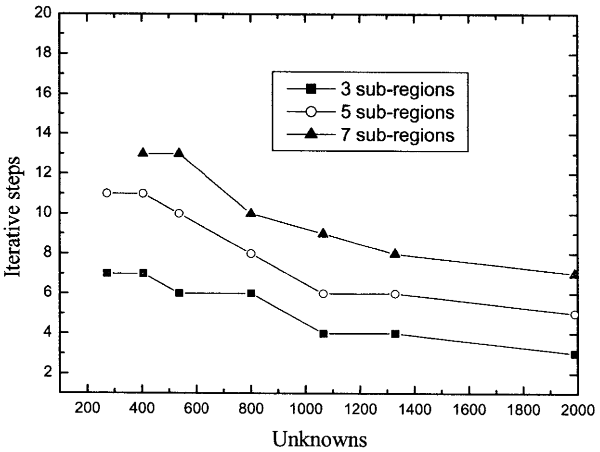

Fig.9 Iterative steps versus the number of unknowns

It is found that the number of iterative steps will nearly keep to a constant for the number of unknown greater than 1200. So the total computational time T can be approximatively expressed by $T \approx \alpha K(M)^3+\beta L(N)^2$. Where, K is number of sub-regions. $M=N/K$. L is the number of iterative steps. From the above analysis $L \propto (N)^r$, $r<0$. Therefore the CPU time can be further expressed by $T \approx \alpha K(M)^3+\beta(N)^{2+r}$.The first term is for evaluating $[Z_{ll}]^{-1}$, the second term is for iterating process, and the α, β are constants depending on the computer performance.

4. Conclusion

In this research, multi-region iterative method has been effectively applied to dielectric body problem on basis of volume integral equation. The numerical results show that the computational CPU time can be reduced significantly if the multi-region iterative method is applied.

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