Three-Dimensional Electromagnetic Scattering Analysis Using Constrained Interpolation Profile Method

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SUMMARY A characteristic-based constrained interpolation profile (CIP) method for solving three-dimensional, time-dependent Maxwell’s equations is successfully developed. It is utilized to solve one-dimensional wave equations in the formulation of the Maxwell’s equations. Calculation procedure of the CIP method for three-dimensional scattering analysis is described in details. Update equations for boundary conditions of a perfectly conducting (PEC) interface and a dielectric interface are formulated and obtained in explicit forms. Numerical analyses of electromagnetic scatterings of PEC sphere, dielectric sphere and PEC cube are performed and the scattering coefficient is calculated and compared with the Mie’s analytic results. As a result, the scattering coefficients show good agreement with the Mie’s results, which demonstrates the validity of the CIP method and the formulated update equations. It is also shown that the phase of the scattering coefficients determined by the CIP method are slightly more accurate than that of the FDTD method.

key words: electromagnetic scattering analysis, scattering cross section, CIP method, FDTD method, Mie

1. Introduction

The field of computational electromagnetics (CEM) has been significantly broadened by the pioneer work of Yee, when the time-dependent Maxwell’s equations are solved numerically in isotropic medium [1]. Yee’s algorithm has been called the finite-difference time-domain (FDTD) method and it has a second-order accuracy in temporal and spatial domains [2]. Since the FDTD method is relatively simple to implement, efficient and robust, so it has been widely applied to many types of electromagnetic problems, and also implemented in many kinds of commercial softwares nowadays. When the FDTD method is used to analyze a problem using a time step size restricted by the Courant-Friedrichs-Lewy (CFL) stability condition, it was shown analytically that there is no dissipative error in the solution obtained by the FDTD method [2]. However, the FDTD method suffers from numerical dispersion or anisotropy, i.e. numerical propagation velocity of electromagnetic wave depends on cell size, time step size, and direction of propagation. This anisotropy causes an accumulative phase error and restricts the analysis solution to the Rayleigh or resonance region where electrical length is in the order of a few wavelengths.

In previous studies, a numerical method, which was developed from a characteristic-based scheme for solving Euler equations in the field of computational fluid dynamics (CFD), has been applied for solving the time-domain Maxwell’s equations [3]–[7]. The coefficient matrices in the Maxwell’s equations are diagonalized in each spatial direction to obtain one-dimensional Riemann formulation for both Cartesian coordinate system and general curvilinear system. Then, a windward difference formulation, i.e. forward or backward difference method, has been used to solve the one-dimensional Riemann problems derived from the eigenvalue analysis and it was shown that the numerical stability can be improved by using the characteristic-based method [6], [8]. In addition, analysis accuracy can be improved by using high-order interpolation or extrapolation technique.

Recently, an application of the characteristic-based constrained interpolation profile (CIP) method to the field of CEM was proposed by Yabe et al. and since then it has been continuously received an attention from researchers [9], [10]. The CIP method is a kind of high-order interpolation method which can accurately solve the hyperbolic equations with third-order accuracy in spatial domain and provides a solution with a small phase error [11].

The CIP method has several advantages over the FDTD method. First, the CIP method can suppress reflected waves from the truncated region by simply forbidding the incoming wave propagation from the far region, whereas the absorbing boundary condition is required in the FDTD method and it actually relates to the complicated formulation of the Maxwell’s equations. This attribute of the CIP method is inherited from the characteristic formulation, which can eliminate the reflected waves completely, if one of the transformed coordinates is aligned with the direction of the electromagnetic waves. Second, the formulation for the CIP method gives a directional biased discretization in spatial domain, which exhibits the physics in directional propagation in accordance with the sign of the eigenvalues, and provides a more stable scheme than a central differencing scheme [4]. Third, phase characteristic of the CIP method shows a better performance than the FDTD method over a broad range of frequencies as shown in [12]. Last, the CIP method allows a waveform which changes abruptly with respect to time, e.g. a rectangular pulse. In spite of many advantages of the CIP method, it also encounters two difficulties. First, since the calculation using the CIP method is the two-step scheme, i.e. the interpolation and the field updating for the field variables and their differential field variables, in
each time step, the computation time of the CIP method is more expensive than the FDTD method, which directly update each field variable with an explicit equation. Moreover, the CIP method uses more memory than the FDTD method because the derivative values of the each field component must be also stored in the memory. Second, the CIP method exhibits a dissipation error after a long-time propagation which is one of the disadvantages of the characteristic-based method.

In [12], it has been shown that the CIP method provides higher accuracy than the FDTD method under the condition of identical cell size of analysis by calculating the electromagnetic field radiated from uniform line current source. However, a three-dimensional scattering analyses of perfectly electric conducting (PEC) and dielectric objects using the CIP method have not been reported so far.

This paper proposes an approach utilizing the CIP method to solve the three-dimensional EM scattering problem and shows algorithms that enable the PEC and the dielectric objects to be modeled into the analysis region. The scattering cross sections (SCS) of PEC sphere and dielectric are shown in Sect. 4, and followed by our conclusion procedure for the three-dimensional CIP method in details.

The rest of the paper is organized as follows. Section 2 reviews the formulation of the Maxwell’s equations in Cartesian coordinate system and shows the calculation procedure for the three-dimensional CIP method in details. Section 3 shows the treatment of boundary conditions for PEC and dielectric interfaces and also boundary conditions at truncated region in the CIP method. Finally, analysis models and numerical results of the SCSs of various objects are shown in Sect. 4, and followed by our conclusion in Sect. 5.

2. Formulation of Maxwell’s Equations

The time-dependent Maxwell’s equations in isotropic medium are expressed as

\[
\frac{\partial (\varepsilon \mathbf{E})}{\partial t} = \nabla \times \mathbf{H} - \mathbf{J},
\]

\[
\frac{\partial (\mu \mathbf{H})}{\partial t} = -\nabla \times \mathbf{E},
\]

where \( \mathbf{E} \) and \( \mathbf{H} \) are the electric and magnetic field strength vectors in volts per meter and amperes per meter, respectively, and \( \mathbf{J} \) is the current density in amperes per square meter. \( \varepsilon \) and \( \mu \) are the permittivity and the permeability of the medium, respectively.

The Maxwell’s equations in free space can be expressed in the conservative form in Cartesian coordinate system as

\[
\frac{\partial \mathbf{W}}{\partial t} + \frac{\partial \mathbf{F}_x}{\partial x} + \frac{\partial \mathbf{F}_y}{\partial y} + \frac{\partial \mathbf{F}_z}{\partial z} = 0,
\]

where \( \mathbf{F}_x = A \mathbf{W}, \mathbf{F}_y = B \mathbf{W}, \) and \( \mathbf{F}_z = C \mathbf{W}, \mathbf{W} = (E_x E_y E_z H_x H_y H_z)^T, \) where superscript \( T \) denotes transpose matrix. The coefficient matrices \( A, B, \) and \( C \) are given by

\[
A = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & \varepsilon^{-1} \\
0 & 0 & 0 & 0 & 0 & -\varepsilon^{-1} \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & \mu^{-1} & 0 & 0 & 0 & 0 \\
0 & 0 & \mu^{-1} & 0 & 0 & 0
\end{bmatrix},
\]

\[
B = \begin{bmatrix}
0 & 0 & 0 & 0 & -\varepsilon^{-1} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & \mu^{-1} & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},
\]

\[
C = \begin{bmatrix}
0 & 0 & 0 & 0 & 0 & \varepsilon^{-1} \\
0 & 0 & 0 & 0 & -\varepsilon^{-1} & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix},
\]

Eigenvalues of coefficient matrices \( A, B \) and \( C \) are found to be zero and \( \pm c \), where \( c \) denotes the velocity of the electromagnetic wave in medium. The eigenvalues have multiplicities and hence the corresponding eigenvectors are not unique. However, in Cartesian coordinate system, linearly independent equations of the system still have been found by the orthogonalization of the coefficient matrices. When the similar matrices of diagonalization are known, the coefficient matrices can be diagonalized individually, and by using a straightforward matrix multiplication, the diagonalized matrices are expressed as

\[
D_x = S_x^{-1} A S_x, \quad D_y = S_y^{-1} B S_y, \quad D_z = S_z^{-1} C S_z,
\]

where diagonal elements of \( D_x, D_y \) and \( D_z \) are equal to the eigenvalues of \( A, B \) and \( C \), respectively, i.e.

\[
\text{Diag}(D_m) = \{c, c, -c, -c, 0, 0\}, \quad m = x, y, z.
\]

\( S_m \) in (7) denotes a non-singular similar matrix composed of eigenvectors as a column vector and \( S_m^{-1} \) is its inverse matrix. The similar matrices and its inverse matrices associated with the coefficient matrices \( A, B \) and \( C \) are given in [3] and they will be omitted here. Since each coefficient matrix can only be diagonalized in \( x, y, \) and \( z \) axes individually, the three-dimensional problem is split into three one-dimensional systems:

\[
\frac{\partial \mathbf{W}}{\partial t} + \frac{\partial (A \mathbf{W})}{\partial x} = 0, \quad (\mathbf{W}^n \rightarrow \mathbf{W}^*),
\]

\[
\frac{\partial \mathbf{W}}{\partial t} + \frac{\partial (B \mathbf{W})}{\partial y} = 0, \quad (\mathbf{W}^* \rightarrow \mathbf{W}^{**}),
\]

\[
\frac{\partial \mathbf{W}}{\partial t} + \frac{\partial (C \mathbf{W})}{\partial z} = 0, \quad (\mathbf{W}^{**} \rightarrow \mathbf{W}^{n+1}),
\]

where, \( \mathbf{W}^n \) is the field at a time step index \( n \) and \( \mathbf{W}^* \) represents the field after propagating along the \( x \) direction. \( \mathbf{W}^{**} \)
represents the field after propagating along the x and y directions. W^{n+1} represents the field after propagating along the x, y, and z directions as shown in Fig. 1. Substituting (7) into (9), we obtain
\[
\partial W \over \partial t + \partial (S_x D_x S_x^{-1} W) \over \partial x = 0. \tag{12}
\]
The similar matrix of the diagonalization and the eigenvalues are brought out of the differential operator to form one-dimensional hyperbolic equations which can be solved by the CIP method. First, we multiply the inverse of the similar matrix from the left side to give
\[
\partial (S_x^{-1} W) \over \partial t + D_x \partial (S_x^{-1} W) \over \partial x = 0, \tag{13}
\]
and for y and z direction, we obtain
\[
\partial (S_y^{-1} W) \over \partial t + D_y \partial (S_y^{-1} W) \over \partial y = 0, \tag{14}
\]
\[
\partial (S_z^{-1} W) \over \partial t + D_z \partial (S_z^{-1} W) \over \partial z = 0. \tag{15}
\]
The equations in (13), (14), and (15) are simply the one-dimensional wave equations and S_x^{-1} W, S_y^{-1} W and S_z^{-1} W are the Riemann invariants or the characteristic variables which hold a constant value along a trajectory in both time and space, with a slope defined by corresponding eigenvalues. Since every equation is completely uncoupled each other, the system of equations can be solved individually as one-dimensional problem, and the CIP method is applied here to solve the equations of the system. The details of the CIP method are described in [10] and will be omitted here.

It should be noted that the calculation procedure is different from previous studies which use the finite-difference or the finite volume method to solve the equations [4, 5]. The characteristic variables are advected forward or backward depending on the associated sign of its eigenvalues in the formulation. The equations for each characteristic variable in each propagation direction can be written as follows. For x direction,
\[
L_{x+}^1 : \left( \frac{\partial}{\partial t} + c \frac{\partial}{\partial x} \right) \left( H_x + \frac{E_y}{\eta} \right) = 0, \tag{16}
\]
\[
L_{x-}^1 : \left( \frac{\partial}{\partial t} - c \frac{\partial}{\partial x} \right) \left( H_x - \frac{E_y}{\eta} \right) = 0, \tag{17}
\]
\[
L_{x+}^2 : \left( \frac{\partial}{\partial t} + c \frac{\partial}{\partial x} \right) \left( H_x - \frac{E_y}{\eta} \right) = 0, \tag{18}
\]
\[
L_{x-}^2 : \left( \frac{\partial}{\partial t} - c \frac{\partial}{\partial x} \right) \left( H_x + \frac{E_y}{\eta} \right) = 0, \tag{19}
\]
for y direction,
\[
L_{y+}^1 : \left( \frac{\partial}{\partial t} + c \frac{\partial}{\partial y} \right) \left( H_y + \frac{E_x}{\eta} \right) = 0, \tag{20}
\]
\[
L_{y-}^1 : \left( \frac{\partial}{\partial t} - c \frac{\partial}{\partial y} \right) \left( H_y - \frac{E_x}{\eta} \right) = 0, \tag{21}
\]
\[
L_{y+}^2 : \left( \frac{\partial}{\partial t} + c \frac{\partial}{\partial y} \right) \left( H_y - \frac{E_x}{\eta} \right) = 0, \tag{22}
\]
\[
L_{y-}^2 : \left( \frac{\partial}{\partial t} - c \frac{\partial}{\partial y} \right) \left( H_y + \frac{E_x}{\eta} \right) = 0, \tag{23}
\]
and for z direction,
\[
L_{z+}^1 : \left( \frac{\partial}{\partial t} + c \frac{\partial}{\partial z} \right) \left( H_z + \frac{E_y}{\eta} \right) = 0, \tag{24}
\]
\[
L_{z-}^1 : \left( \frac{\partial}{\partial t} - c \frac{\partial}{\partial z} \right) \left( H_z - \frac{E_y}{\eta} \right) = 0, \tag{25}
\]
\[
L_{z+}^2 : \left( \frac{\partial}{\partial t} + c \frac{\partial}{\partial z} \right) \left( H_z - \frac{E_y}{\eta} \right) = 0, \tag{26}
\]
\[
L_{z-}^2 : \left( \frac{\partial}{\partial t} - c \frac{\partial}{\partial z} \right) \left( H_z + \frac{E_y}{\eta} \right) = 0, \tag{27}
\]
where, L_{ma}^{\pm} denotes the differential operators on m axis (m=x, y, z). Superscript s (=1,2) denotes an index of independent linear equations in each propagation direction and we will use this notation to describe the calculation procedure. The signs ± represent the forward or backward propagations of electromagnetic field in the associated m direction. Assuming that the medium is homogeneous and the propagation velocity c is constant at every points in analysis region, a set of differential equations in each direction can be found directly by differentiation with respect to x, y, and z axes. The system of differential equations have the similar form with the above equations, e.g., the differentiation of (16) with respect to x axis gives,
\[
\left( \frac{\partial}{\partial t} + c \frac{\partial}{\partial x} \right) \left( \partial_x H_x + \frac{\partial_x E_y}{\eta} \right) = 0. \tag{28}
\]
Next we consider H_x - E_y pairs of (16) and (17). Another pairs of the field components can be formulated in the same way. Figure 2 illustrates the concept of characteristic method for the field propagation. The field components H_x and E_y at the time step index n propagate forward or backward along the x axis, corresponding to the sign of velocity c. The forward propagating field components, which are calculated from the field values and its derivatives at grid points x_i and x_i-1 by using the CIP method, are denoted by H_{x+}^{m+} and E_{y+}^{m+}, respectively. The backward propagating field components calculated by using the CIP method are denoted by H_{x-}^{m-} and E_{y-}^{m-}. After the field components propagating into the left and right region as shown in Fig. 2 are found, the
fields at the grid point $x_i$ are calculated by using the invariant of characteristic variables as

$$H_z^n + \frac{E_z^n}{\eta} = H_z^{n+1} + \frac{E_z^{n+1}}{\eta}, \quad (29)$$

$$H_y^n - \frac{E_y^n}{\eta} = H_y^{n+1} - \frac{E_y^{n+1}}{\eta}. \quad (30)$$

Solving (29) and (30) gives update equations for electric and magnetic fields at the grid point $x_i$ in explicit forms as

$$E_y^n = \frac{1}{2} \left\{ E_y^n + E_y^{n+1} - \eta H_y^{n+1} - \eta H_y^n \right\}, \quad (31)$$

$$H_y^n = \frac{1}{2} \left\{ \frac{E_y^n}{\eta} - \frac{E_y^{n+1}}{\eta} + H_y^{n+1} + H_y^n \right\}, \quad (32)$$

where $E_y^n$ and $H_y^n$ are the field components located at $x_i + c\Delta t$ with the time step index, $n$, and propagating toward the grid point, $x_i$. Similarly, $E_y^n$ and $H_y^n$ are the field components located at $x_i - c\Delta t$ and propagating toward the same grid point, $x_i$. Superscript $*$ is used to represent the fields after propagating along the $x$ axis as same as in Eq. (9). The equations in (31) and (32) are the update equations for the differential operator $L^n_x$. The update equations for the differential operators $L^n_y$, $L^n_z$, $L^n_{xy}$, and $L^n_{yz}$ can also be determined in the same way and can be expressed as follows. For $L^n_y$ operator,

$$E_y^n = \frac{1}{2} \left\{ E_y^n + E_y^{n+1} - \eta H_y^{n+1} + \eta H_y^n \right\}, \quad (33)$$

$$H_y^n = \frac{1}{2} \left\{ \frac{E_y^n}{\eta} - \frac{E_y^{n+1}}{\eta} + H_y^{n+1} - H_y^n \right\}, \quad (34)$$

for $L^n_z$ operator,

$$E_z^n = \frac{1}{2} \left\{ E_z^n + E_z^{n+1} - \eta H_z^{n+1} + \eta H_z^n \right\},$$

$$H_z^n = \frac{1}{2} \left\{ \frac{E_z^n}{\eta} - \frac{E_z^{n+1}}{\eta} + H_z^{n+1} - H_z^n \right\}, \quad (36)$$

for $L^n_{xy}$ operator,

$$E_y^n = \frac{1}{2} \left\{ E_y^n + E_y^{n+1} - \eta H_y^{n+1} + \eta H_y^n \right\}, \quad (37)$$

$$H_y^n = \frac{1}{2} \left\{ \frac{E_y^n}{\eta} - \frac{E_y^{n+1}}{\eta} + H_y^{n+1} - H_y^n \right\},$$

for $L^n_{yz}$ operator,

$$H_z^n = \frac{1}{2} \left\{ E_z^n + E_z^{n+1} - \eta H_z^{n+1} - \eta H_z^n \right\}, \quad (38)$$

for $L^n_{xy}$ operator,

$$H_z^n = \frac{1}{2} \left\{ E_z^n + E_z^{n+1} + \eta H_z^{n+1} + \eta H_z^n \right\}, \quad (39)$$

$$H_y^n = \frac{1}{2} \left\{ \frac{E_y^n}{\eta} - \frac{E_y^{n+1}}{\eta} + H_y^{n+1} - H_y^n \right\}, \quad (40)$$

and for $L^n_{yz}$ operator,

$$E_y^n = \frac{1}{2} \left\{ E_y^n + E_y^{n+1} + \eta H_y^{n+1} + \eta H_y^n \right\}, \quad (41)$$

$$H_y^n = \frac{1}{2} \left\{ \frac{E_y^n}{\eta} - \frac{E_y^{n+1}}{\eta} + H_y^{n+1} - H_y^n \right\}, \quad (42)$$

where superscript $\ast$ is used to describe the field components after propagating along the $x$ and $y$ axes. The update equations formulated above are equivalent to the determination of $\mathbf{W}$ by multiplying the similar matrix $S_n$ with $\mathbf{S}^{-1} \mathbf{W}$. It is interesting to note here that there is no field component aligned in the $x$, $y$, and $z$ axes propagating along the same $x$, $y$, and $z$ directions because they are corresponding to the associated eigenvalue of zero. The update equations formulated using the differential equations are determined in the same way, e.g., from (28) we obtain,

$$\partial_t E_y^n = \frac{1}{2} \left\{ \partial_t E_y^n + \partial_t E_y^{n+1} + \eta \partial_y H_y^{n+1} + \eta \partial_y H_y^n \right\}, \quad (43)$$

$$\partial_t H_y^n = \frac{1}{2} \left\{ \partial_t H_y^n + \partial_t E_y^{n+1} + \eta \partial_y H_y^{n+1} + \eta \partial_y H_y^n \right\}. \quad (44)$$

In addition to the formulation of the wave equations differentiated with respect to the $x$ axis, the differentiations with respect to the axis perpendicular to the propagation direction, e.g., $\partial_y E_y$ and $\partial_y H_y$, must be also computed. Here we apply the first-order upwind scheme to those calculations as, for $c > 0$,

$$\partial_y E_y^n(k) = \partial_z E_y^n(k) - \frac{c \Delta t}{\Delta y} \left( \partial_y E_y^n(k) - \partial_y E_y^n(k - 1) \right), \quad (45)$$

$$\partial_y H_y^n(k) = \partial_z H_y^n(k) - \frac{c \Delta t}{\Delta y} \left( \partial_y H_y^n(k) - \partial_y H_y^n(k - 1) \right), \quad (46)$$

and for $c < 0$,

$$\partial_y E_y^n(k) = \partial_z E_y^n(k) + \frac{c \Delta t}{\Delta y} \left( \partial_y E_y^n(k) - \partial_y E_y^n(k + 1) \right), \quad (47)$$

$$\partial_y H_y^n(k) = \partial_z H_y^n(k) + \frac{c \Delta t}{\Delta y} \left( \partial_y H_y^n(k) - \partial_y H_y^n(k + 1) \right), \quad (48)$$

where $\Delta t$ is the time step size, $\Delta z$ is the cell size, and $k$ is the grid index in $z$ axis. Then, by using (45)–(48), $\partial_y E_y$ and $\partial_y H_y$ can be determined from equations with the same form.
as (43) and (44). \( \partial_y E_y \) and \( \partial_y H_z \) can be computed in the same manner. This kind of the CIP method is called “type-M CIP method” [13], which requires less memory compared to another type of the CIP method.

A cyclic sequence of one-dimensional operators is applied to maintain the symmetry of the solutions. The fields at the next time step \( n + 2 \) can be then expressed by using the notation of the differential operators as,

\[
\begin{align*}
W^{n+1} &= L_z L_y L_x W^n, \\
W^{n+2} &= L_z L_y L_x W^{n+1}.
\end{align*}
\]

(49)

(50)

The same sequence is also applied to the system equations of the differentiated field components.

3. Boundary Conditions in the CIP Method

In this section, boundary conditions for perfect conducting (PEC) interface and those between two media, and a boundary condition for suppression of reflected waves at truncated region are described.

3.1 Treatment of the Fields Close to the PEC Interface

Figure 3 shows an interface between free space and PEC object in one-dimensional domain. An incident plane wave is coming from right region with the velocity \( c \). The PEC boundary is defined at a middle plane between two grid points, \( x_i \) and \( x_i-1 \). The electric field at the PEC boundary satisfies the condition:

\[
\hat{n} \times \mathbf{E} = 0,
\]

(51)

where \( \mathbf{E} \) is a total electric field and \( \hat{n} \) is a surface outward normal vector. The fields close to the interface can be determined by creating an image of the electromagnetic field at \( x_i-1 \), i.e.

\[
\begin{align*}
E_y^r(x_i-1) &= -E_y^t(x_i), \\
H_z^r(x_i-1) &= H_z^t(x_i),
\end{align*}
\]

(52)

(53)

where, subscript \( r \) denotes the tangential field component. After the substitution of the field value at \( x_i \) is done, the CIP method is then applied to determine the field values at the next time step index, \( n + 1 \), according to the update Eqs. (31)–(42).

3.2 Treatment of the Fields Close to the Interface between Two Different Media

Figure 4 shows the interface between two different media in one-dimensional domain. Only the formulation of \( H_z - E_y \) pairs is described here and another pairs of the field components can be formulated in the same manner. The invariant of characteristic variables in two media is given by

\[
\begin{align*}
H_z^+ + \frac{E_y^+}{\eta_1} &= H_y^+ + \frac{E_y^+}{\eta_1}, \\
H_z^- - \frac{E_y^-}{\eta_2} &= H_y^- - \frac{E_y^-}{\eta_2},
\end{align*}
\]

(54)

(55)

By solving (54) and (55) for \( E_y^\pm \) and \( H_z^\pm \), the update equations for electromagnetic fields at the dielectric interface are obtained and given by

\[
\begin{align*}
E_y^\pm &= \frac{\eta_1 \eta_2}{\eta_1 + \eta_2} \left\{ \frac{E_y^+}{\eta_1} + \frac{E_y^-}{\eta_2} + H_z^+ - H_z^-, \right\}, \\
H_z^\pm &= \frac{1}{\eta_1 + \eta_2} \left\{ E_y^+ - E_y^- + \eta_1 H_z^+ + \eta_2 H_z^- \right\},
\end{align*}
\]

(56)

(57)

where \( \eta_1, \eta_2 \) are the intrinsic impedances in media #1 and #2, respectively. It should be noted that the update equations for the differential field values have the similar form as (56) and (57).

3.3 Boundary Conditions at Truncated Region

Since the CIP method calculates the field translation in each propagation direction individually, the reflection at the truncated region can be suppressed using the information of the propagation direction. Only incoming waves from the far field are set to be the null value at the truncated region as

\[
\begin{align*}
W_{y^+}^{n}(0, j, k) &= W_{y^-}^{n}(NX - 1, j, k) = 0, \\
W_{y^+}^{n}(i, 0, k) &= W_{y^-}^{n}(i, NY - 1, k) = 0, \\
W_{y^+}^{n}(i, j, 0) &= W_{y^-}^{n}(i, j, NZ - 1) = 0,
\end{align*}
\]

(58)

(59)

(60)

where \( W_{y^+}^{n}(i, j, k) \) and \( W_{y^-}^{n}(i, j, k) \) are the field component.
vectors at the grid location index, \((i, j, k)\), and the time step index, \(n\), which are propagating forward and backward, respectively, along the \(m\) \((m = x, y, z)\) axis from the far region into the analysis region. NX, NY, and NZ are total numbers of the discretization of the analysis region in the \(x\), \(y\), and \(z\) axes, respectively. This boundary condition is well-posed only when the propagation direction of electromagnetic wave is aligned with Cartesian coordinate system. When the propagation direction is not aligned with Cartesian coordinate system, the exact no-reflection boundary condition is degraded to an approximation and it behaves like the first-order Mur’s absorbing boundary condition (ABC). This is because only the electromagnetic waves propagating along the direction normal to the truncated boundary surface are transmitted without reflections as same as the first-order Mur’s ABC [14].

4. Numerical Results

In order to confirm the validity of the proposed algorithms, the bistatic scattering cross sections (BSCS) of PEC sphere, dielectric sphere, and PEC cube are calculated using the CIP method and compared with the results calculated by the Mie’s series and the FDTD method.

Figure 5 shows analysis models of PEC sphere and dielectric sphere for the CIP method. The radius of the spheres is \(a = 5\) cm, the size of analysis region is \(30\) cm \(\times\) \(30\) cm \(\times\) \(30\) cm, and the cell sizes are \(1\) mm \(\times\) \(1\) mm \(\times\) \(1\) mm in \(x\), \(y\), and \(z\) directions, respectively. The incident plane wave is assumed to be a rectangular pulse wave with a pulse width of \(5\Delta\), where \(\Delta\) denotes the cell size in each axis. The information of the propagation is incorporated to suppress the reflected field at the outermost boundary as describe in Sect. 3.3. The reason for using the rectangular pulse as an incident plane wave in the CIP method is that because the interpolation using third-order polynomials is applied in the CIP method, so that the CIP method can represent the rectangular waveform more accurately than the Gaussian waveform which is actually used in the FDTD method. The CIP method can treat the waveform which changes abruptly with respect to time. This is also one of the advantages of the CIP method.

Figure 6 shows the analysis model used for the FDTD analysis. The radius of the spheres, the size of analysis region and the cell sizes are the same to those in the CIP model. Total-field/scattered-field (TF/SF) boundary [2] is applied to illuminate the plane wave into the analysis region. A perfectly matched layer (PML) is also used to suppress the reflection of electromagnetic waves at the outermost boundary. Gaussian waveform with a pulse width of \(\tau_0 = 133\) nsec. and an attenuation constant of \(\alpha = 16/\tau_0^2\) is employed as an incident plane wave.

The Courant-Friedrich-Levy (CFL) number is defined as \(\text{CFL} = c\Delta t/\Delta\) for the CIP method, where \(\Delta t\) is the time step size. Since the maximum time step size is restricted by the Courant stability condition [2], so that the maximum CFL number for the FDTD model is about \(1/\sqrt{3} = 0.577\) but it is unity for the CIP model, which means that the CIP method requires less time steps compared with the FDTD method. Parameters for the numerical analyses are summarized in Table 1.

In order to calculate the BSCS evaluated by the far-zone field, the equivalent electric surface currents, \(J_s\), and magnetic surface currents, \(M_s\), located on a cubic surface encompassing the sphere are used as shown in Figs. 5 and 6. Since the outermost boundary of the CIP model behaves like the first-order Mur’s ABC in the FDTD method, the re-

<table>
<thead>
<tr>
<th>Table 1</th>
<th>Parameters for analysis</th>
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</thead>
<tbody>
<tr>
<td>Analysis region</td>
<td>300x300x300 cells</td>
</tr>
<tr>
<td>Cell size (\Delta x, \Delta y, \Delta z)</td>
<td>1 mm</td>
</tr>
<tr>
<td>Analysis frequency</td>
<td>5 GHz, 10 GHz</td>
</tr>
<tr>
<td>CFL number</td>
<td>1</td>
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<tr>
<td>Incident pulse</td>
<td>rectangular pulse</td>
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<tr>
<td>ABC</td>
<td>None</td>
</tr>
<tr>
<td>Number of time steps</td>
<td>500 (PEC case)</td>
</tr>
</tbody>
</table>
flected wave will appear and degrade accuracy of the analysis when the incident angle of electromagnetic fields is not normal to the boundary. To avoid this problem, the equivalent current surfaces are placed far enough from the outermost boundary, i.e. 80 cells from the outermost boundary in our simulation. Finally, the bistatic scattering coefficient is calculated by the following equation.

\[ \mu_s = \lim_{r \to \infty} 2 \sqrt{\frac{\pi r}{E_{inc}}} \]  \hspace{1cm} (61)  

where, \( E_{inc} \) represents the incident electric field and \( E' \) represent the scattered electric field.

Figures 7 and 8 show the BSCS \( \sigma = |\mu_s|^2 \) and the phase of the scattering coefficient of the PEC sphere at 5 GHz by using the CIP and the FDTD methods. The analysis results at 10 GHz are also shown in Figs. 9 and 10. The results at both 5 and 10 GHz are obtained simultaneously via discrete Fourier transform of the electric and magnetic fields on the closed surface. The scattering coefficient at another frequencies can also be calculated easily by one single-pulse excitation, which is one of the advantages of the time-domain method. The exact solutions for the BSCS and the phase of the scattering coefficient obtained by the Mie’s series are also plotted by the solid line in these figures. The FDTD results are plotted by the dashed line. The results obtained by both the CIP and the FDTD methods show good agreements with the Mie’s results for PEC sphere case. However, the reflected waves from the outermost boundary are still observed and affect the bistatic scattering coefficient amplitude as \( \theta = 30^\circ - 60^\circ \) in the CIP model.

As a second example of the scattering analysis using the CIP method, the scattering coefficient of the dielectric sphere with the radius of \( a = 5 \text{ cm} \) is calculated. Figures 11 and 12 show the BSCS and the phase of the scattering coefficient of the dielectric sphere at 5 GHz by using the CIP and the FDTD methods. The results at 10 GHz are also shown in Figs. 13 and 14. The results obtained by both the CIP and the FDTD methods show good agreements with the Mie’s results for dielectric sphere case. However, the reflected waves from the outermost boundary are still observed and affect the bistatic scattering coefficient amplitude as \( \theta = 30^\circ - 60^\circ \) in the CIP model.
noticed that the phase of scattering coefficient obtained by the CIP method is slightly more accurate than that of the FDTD method for $\theta$ close to 0°, 180°. This is due to the fact that the FDTD method suffers from the phase error caused by the anisotropy especially at $\theta = 0^\circ$ and 180°, where the electromagnetic wave penetrates into dielectric material. Although the decrease in the accuracy of the FDTD analysis is not so important in the present model, it could be more serious when the analysis domain is very large and the information of propagation phase is very important. This observation shows that the CIP method could be an alternative method for a large analysis model.

As a third example, the CIP method was applied to the numerical analysis of the electromagnetic scattering of a PEC cube with one-side length of 5 cm. The model for the analysis is the same to Figs. 5 and 6 except that the sphere is replaced with the cube. Since the exact solution is not available for the PEC cube case, the method of moment (MoM) and the FDTD method was employed and their results are compared with the CIP results. The MoM analysis was performed by a commercial electromagnetic analysis simulation software called FEKO. The BSCS and the phase of scattering coefficient for the PEC cube are shown in Figs. 15 and 16. It can be seen that the CIP results agree well with
those obtained by the FDTD method and the MoM, which again confirms the validity of CIP method.

Although the CIP method can be used to calculate the SCS of the dielectric sphere with a small permittivity as demonstrated above, the CIP method becomes unstable when dielectric constant becomes higher. As an example, Fig. 17 shows the electric field strength \( E_z \) with respect to the time for the CIP and FDTD method for the case of the dielectric sphere with \( \varepsilon_r = 4 \).

The reason for the instability is that the update equations expressed in (56) and (57) for the interface between two dielectrics is an approximation for the case that spatial changes of the permittivity are small. For example, the differentiation of (16) and (17) gives following equations: for \( L^1_{+} \) operator,

\[
\left( \frac{\partial}{\partial t} + c \frac{\partial}{\partial x} \right) \left( \partial_x E_z + \frac{\partial E_y}{\eta_1} \right) = - \frac{\partial c}{\partial x} \left( H_z + \frac{\partial E_y}{\eta_1} \right),
\]

and for \( L^1_{-} \) operator,

\[
(\frac{\partial}{\partial t} - c \frac{\partial}{\partial x}) \left( \partial_x H_z - \frac{\partial E_y}{\eta_2} \right) = \frac{\partial c}{\partial x} \left( \frac{\partial E_y}{\eta_2} \right). \tag{63}
\]

In our formulations, the spatial derivative of the propagation velocity or \( \frac{\partial c}{\partial x} \) is considered to be small enough to be neglected so that the right-hand term is equal to zero. However, for a high-permittivity dielectric case, this term can not be neglected and it causes the instability in our simulation.

Next, the effect of the permittivity value and the cell size to the stability of the CIP method is demonstrated. Number of the time steps which the solutions are stable for each value of the permittivity is determined from the time-domain plot and the results are summarized in Table 2. From the Table 2, it is seen that when the permittivity of the sphere becomes higher, the CIP method becomes more unstable. Although the instability can be relieved by using a larger cell size, the changing in the cell size also affects accuracy of discretization of the model so that it is impractical in some cases. Improvement of the stability of the CIP method at the interface between dielectrics is also a part of our future works.

Finally, calculation time and memory usage for the CIP and FDTD methods are shown in Table 3. All programs are run on the SX-9 supercomputer in Tohoku University, Japan. From the Table, it is seen that CPU time for the CIP method is longer than that for the FDTD method since the calculation using the CIP method is the two-step scheme as described above. Although the CIP method requires fewer time step, the computations at each step are more expensive than the FDTD method. Moreover the memory usage for the CIP method is about 2 times of the FDTD method because the CIP method needs to store both the field values and the differential field values. The memory usage for the CIP method can be approximately determined from the following equation;

\[
\text{Memory usage} = 24 \times N_X \times N_Y \times N_Z \times 8 + 3 \times N_X \times N_Y \times N_Z \times 8 + 4 \times N_X \times N_Y \times N_Z .
\]

The first term is for the variables of the field components and their differential field components. The second term is for the constants at each grid points (intrinsic impedance, propagation velocity, permittivity of medium). The last term is for material index at each grid point.
5. Conclusion

The characteristic-based CIP method for solving three-dimensional Maxwell’s equations has been developed. The boundary conditions for PEC and dielectric interfaces have been formulated and implemented successfully. The CIP method is then applied to the analysis of the scattering from the PEC sphere and the dielectric sphere and compared with the exact solutions and the FDTD results. It is also applied to the scattering from the PEC cube and compared with the FDTD and the MoM results. The agreement of the present results is fairly good with the exact analytic results, which demonstrates the validity of the CIP method. Although a stability problem is still remained, it has been shown that CIP method can be an alternative numerical technique for solving electromagnetic problems and could broaden the field of the CEM.

Extension of Cartesian grid-based system to the body-fitted coordinate system with the utilization of Jacobian coordinate transformation is remaining work to increase the accuracy.

Acknowledgement

This work was partly supported by Tohoku University 21st Century Center of Excellent (COE) program. Parts of the numerical results in this research were obtained by using supercomputing resources at Information Synergy Center, Tohoku University.

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