PAPER

Analysis of Huge-Scale Periodic Array Antenna Using Impedance Extension Method

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SUMMARY An extreamly large scale periodic array antenna is required for transmitting power from space solar power systems. Analysis of the huge-scale array antenna is important to estimate the radiation property of the array antenna, but a full-wave analysis requires too much computer memory and excessive CPU time. In order to overcome these difficulties, the impedance extension method is proposed as a method of approximate analysis for huge periodic array antennas. From the results of actual gain pattern obtained by the proposed method and its relative error, it is shown that edge effects of a huge-scale array antenna can be ignored in calculating the radiation property.
key words: *Method of Moments(MoM), impedance extension method, huge periodic array antenna*

1. Introduction

Exhaustion of the energy source including fossil fuel has become one of the most serious problems in recent years. As one of the approaches to solve the problem, research on the SSPS (Space Solar Power Systems), which generates the power by solar cells mounted on stationary satellite and transmits the power to the earth with microwave, has attracted considerable attention as described in [1]. In order to transmit the microwave power from the SSPS to the earth, a huge array antenna composed of hundreds of millions of elements is used to obtain a extremely narrow beam width of microwave. Therefore, beam width, actual gain pattern and edge effects of a huge periodic array antenna have to be analyzed for the design of practical use of SSPS.

Many efforts have been made to analyze the electromagnetic property of the periodic array antenna. Ishimaru et al. [2] proposed a finite periodic structure method for the analysis of a finite array antenna and compared the active impedance of an 11×11 element dipole array antenna over a ground plane obtained by the proposed method with exact solution and infinite periodic structure. Hansen and Gammon presented the effects of the active impedance of finite-by-infinite array antenna consisting of dipole elements with a ground plane [3], [4]. They also proposed a Gibbsian model to express the edge effects of the active impedance [5], [6]. It has been also reported that the active impedance is independent of a number of elements and E-plane impedance can be modulated form depending on the radius of elements [8]. Although many results of the active impedance of a periodic array antenna have been reported in these papers, analysis for the actual gain pattern and systematic analysis of the array antenna have not been reported yet.

Meanwhile, many researches of methods for large-scale problem have been carried out. One of these researches is the acceleration of Method of Moments (MoM) [9], [10]. MoM is a popular method to analyze antennas as well as conducting scatterers, but the CPU time and memory are proportional to N^3 and N^2, respectively, when Gaussian elimination is used to solve a $N \times N$ MoM matrix, where N is the number of unknown coefficients in MoM. Therefore, analysis of a periodic array antenna for SSPS which has so many elements as hundreds of millions is almost impossible and it is important to develop a much efficient method to analyze such a huge scale of array antenna. Many approaches to improve MoM have been proposed.

Iterative methods such as Gauss-Seidel method, Successive Over Relaxation (SOR) method and Conjugate Gradient (CG) method were introduced to reduce the CPU time for the analysis of large scale problems [11]–[14]. The CPU time for analysis of large scale problems is reduced to $O(N^2)$ theoretically, when iterative methods are used to solve the MoM matrix. However, the iterative steps of $O(N)$ are usually required to obtain convergent solution in the iterative method. The Fast Multipole Method (FMM) and Multi-Level Fast Multipole Algorithm (MLFMA) have been proposed to reduce the computational complexity of the matrix-vector multiplication in an iterative method [15]–[17]. CG-FMM-FFT method and preconditioning CG-FFT method have also been proposed as a method to analyze a periodic array antenna having ten thousand elements with PC [18]–[20]. Even when these efficient methods are used, it is still impossible to analyze a huge periodic array antenna having hundreds of millions of elements.

In this paper, properties of the active impedance of a periodic array antenna, which were reported in previous papers, are reviewed briefly with numerical analysis by MoM. Impedance extension method, which can be used for the approximate numerical analysis of a huge periodic array antenna using properties of the active impedance, is proposed. Finally, validity of the proposed method is discussed by cal-

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culating the actual gain pattern of one-dimensional H-plane periodic dipole array antenna.

2. Active Impedance

Active impedance of array element of one-dimensional H-plane periodic array antenna composed half-wavelength dipole elements shown in Fig. 1 is analyzed before developing the impedance extension method. In Fig. 1, the number of array elements, the length of each dipole, the array spacing and the height of the array elements are $N_x = 50$, $2l = 0.5\lambda$, $d_x = 0.75\lambda$ and $h = 0.25\lambda$, respectively. Each dipole is divided into 3 dipole segments in MoM analysis. As the feeding amplitude of the voltage of the array element, uniform distribution, 10 dB-tapered and 20 dB-tapered Gaussian distributions shown in Fig. 2 are employed.

Magnitude and phase of the active impedance for in-phase feeding is shown in Fig. 3. The magnitude and phase of the active impedance changes sharply in the edge region, but is almost uniform in the central region as pointed out by papers [3]–[8]. Also, the magnitude and phase of the active impedance is almost independent of distribution of feeding amplitude as noted in [2].

Magnitude of the active impedance for 10 dB taper with varying the values of array spacing d_x and the length of the dipole elements $2l$ is shown in Fig. 4 and Fig. 5, respectively. The magnitude of the active impedance depends on the array spacing and dipole length, but the behavior is similar.

The magnitude of the active impedance for N_x =

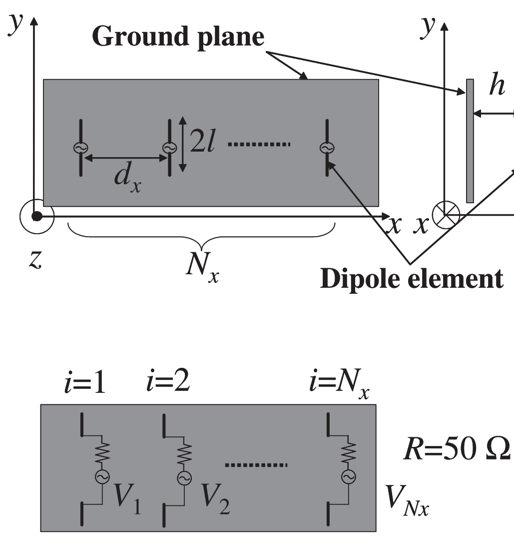

Fig. 1 One-dimensional H-plane periodic dipole array antenna.

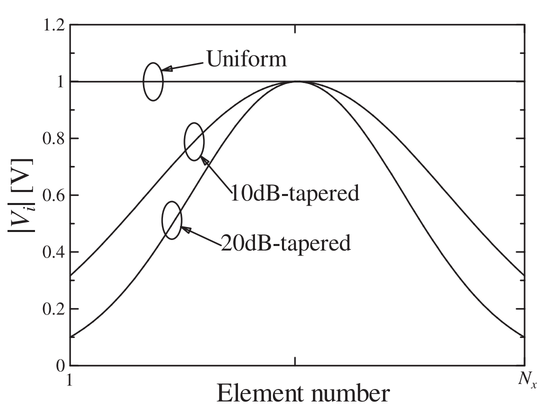

Fig. 2 Taper with Gaussian distribution.

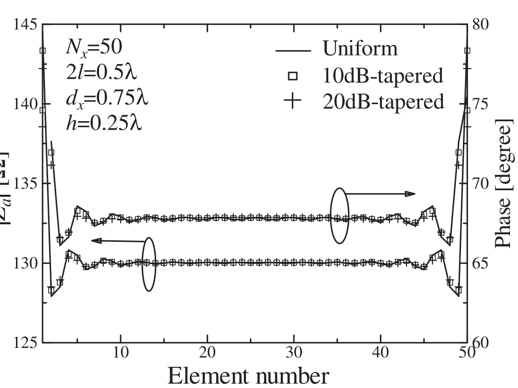

Fig. 3 Active impedance of array antenna by 10 dB, 20 dB taper (magnitude, phase).

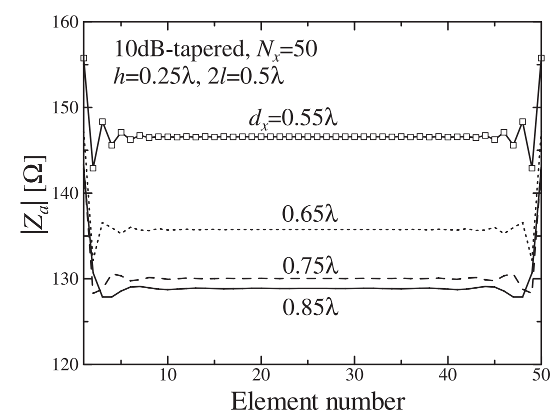

Fig. 4 Active impedance of an array antenna for different d_x ($2l = 0.5\lambda$, magnitude).

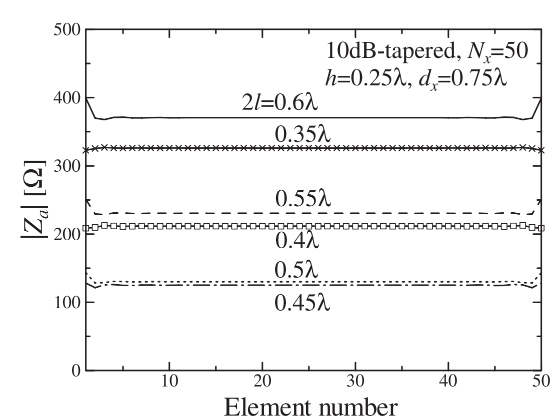

Fig. 5 Active impedance of an array antenna for different $2l$ ($d_x = 0.75\lambda$, magnitude).

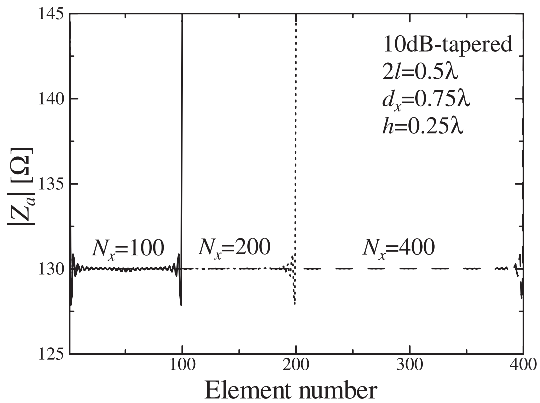

Fig. 6 Active impedance of an array antenna for different N_x ($d_x = 0.75\lambda$, $2l = 0.5\lambda$, magnitude).

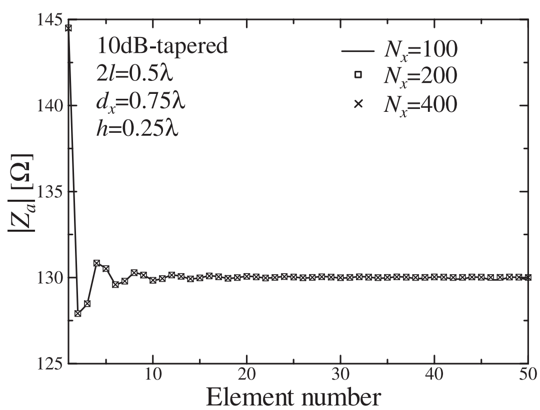

Fig. 7 Active impedance of an array antenna for different N_x ($d_x = 0.75\lambda$, $2l = 0.5\lambda$, magnitude, representation of edge).

100, 200 and 400 is shown in Fig. 6 and Fig. 7. It is noted that the magnitude of the active impedance is almost independent of the number of elements N_x and agree with each other in the edge and central regions. This phenomenon has been already reported in [5], [6].

3. Impedance Extension Method

Numerical results shown in Sect. 2 and previously reported papers [2]–[8] yield following properties of active impedance for a periodic array antenna.

1. Active impedance is almost independent of distribution of feeding voltage.
2. Active impedance changes in the edge region, and is almost uniform in the central region.
3. Active impedance is almost independent of a number of elements, when the number of array elements is larger enough.

From these properties, it is possible to expand the active impedance of a small periodic array antenna (called "small array") having N_x^o elements to a huge periodic array antenna (called "huge array"). The impedance extension method is illustrated in Fig. 8. The values of the active

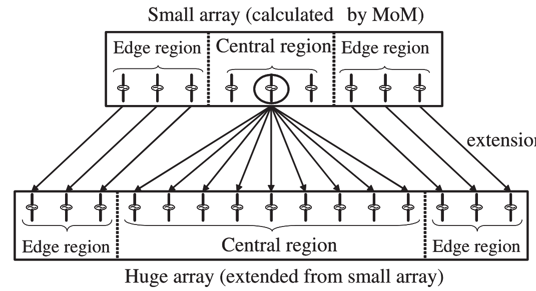

Fig. 8 Introduction of the impedance extension method.

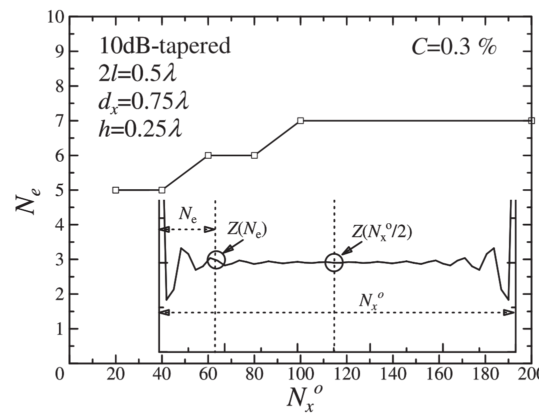

Fig. 9 Number of elements yielding $\Pi_L < 0.3\%$.

impedance of the elements in the edge region of the small array is substituted into those of the elements in the edge region of the huge array. The active impedance of the center element in the small array is substituted into the impedance of all elements of the huge array in the central region. To define the boundary between the edge and the central regions, edge effects are estimated using following equation,

$$\Pi_L(i) = \frac{|Z(i) - Z(N_x^o/2)|}{|Z(N_x^o/2)|} \quad i = 1, ..., N_x^o \tag{1}$$

where $Z(i)$ is the active impedance of ith element. Π_L expresses the difference of the active impedance between ith element and the center element. The element number of the edge region N_e is determined by $\Pi_L(N_e) < C$, where C is a sufficiently small number to define the boundary between the edge and the central regions. The number of the edge elements N_e which yields $\Pi_L < 0.3\%$ is shown in Fig. 9 as a function of the total number of the array elements. From this figure, it is found that $N_e = 7$ is enough for impedance extension from a small array having $N_x^o = 50$. The number of edge elements is dependent on value of C and $C = 0.3\%$ is used in this paper. As shown in Fig. 14, $C = 0.3\%$ (corresponding $N_e = 7$) is the value to ensure the relative error of radiation field smaller than 0.1% as long as N_x is greater than 200. A small value of C will decrease the error of the approximation, while increase the CPU time for numerical analysis.

The impedance extension method is applied to a huge array. Active impedance of a small array having $N_x^o = 50$

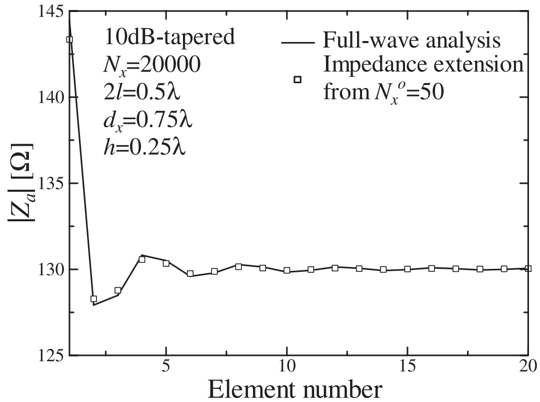

Fig. 10 Comparison of active impedance obtained by impedance extension method and full-wave analysis (absolute value, edge region).

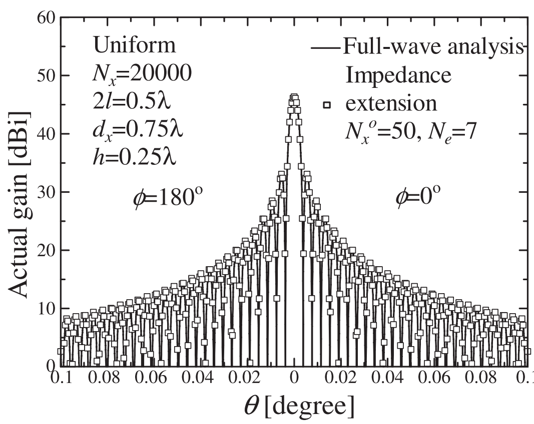

Fig. 11 Actual gain pattern obtained by impedance extension method and full-wave analysis.

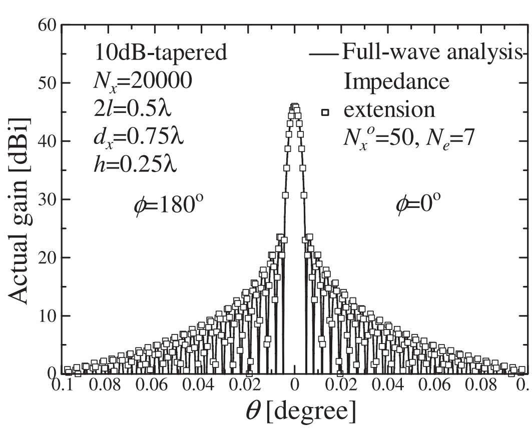

Fig. 12 Actual gain pattern obtained by impedance extension method and full-wave analysis (10 dB tapered distribution).

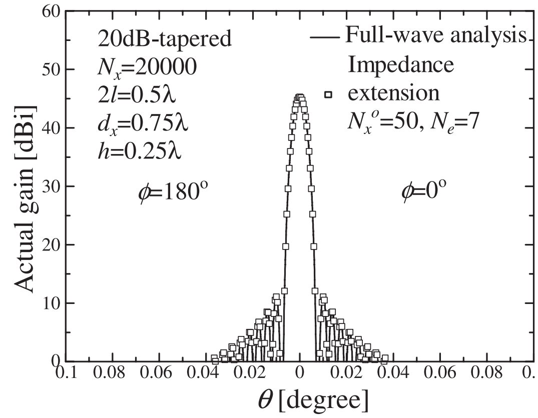

Fig. 13 Actual gain pattern obtained by impedance extension method and full-wave analysis (20 dB tapered distribution).

is extended to a huge array with $N_x = 20000$. Since image method is used to include the effect of ground plane, the total number of elements to be analyzed is 40000. The current distribution of each array element is required to calculate the radiation field, and it is assumed that all elements have the same current distribution to that of the center element in the small array. Using this current distribution, actual gain pattern of the huge array is obtained.

Magnitude of active impedance obtained by the impedance extension method and MoM is shown in Fig. 10. Actual gain pattern of the array antenna which has the uniform and tapered distribution for feeding voltage is shown in Figs. 11–13. It is found that the active impedance and the actual gain by the proposed method agree with those by the full-wave analysis very well.

4. Error of Impedance Extension Method

In the previous section, accuracy of the impedance extension method is discussed through active impedance and actual gain pattern of the array antenna. In this section, relative accuracy of the actual gain pattern obtained by the impedance extension method is evaluated quantitatively. The edge effects on the actual gain of the huge array antenna are also investigated.

Error of the radiation field is defined by the following equation,

$$\varepsilon = \frac{\sqrt{\sum_{i=1}^{P} |E_1(\theta_i) - E_2(\theta_i)|^2}}{\sqrt{\sum_{i=1}^{P} |E_1(\theta_i)|^2}} \qquad (2)$$

where $E_1(\theta_i)$ is the radiation field obtained by MoM, and $E_2(\theta_i)$ is that obtained by the impedance extension method. P is the number of sampling points of θ_i. The relative error of the actual gain pattern obtained by the proposed method can be evaluated using Eq. (2).

Figure 14 shows the error defined by Eq. (2) as a function of the number of the array elements N_x. Error of the radiation field obtained by ignoring the edge effects and using the value of driving point current of the center element in the small array for all array elements is also plotted in Fig. 14 as "Infinite array." Errors of two methods are almost the same and very small as shown in this figure. Error becomes small

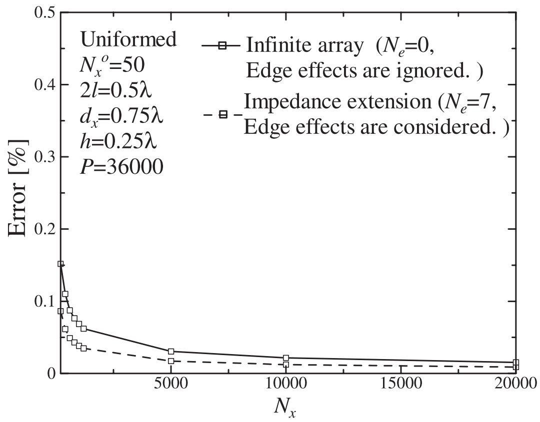

Fig. 14 Error of radiation field obtained by impedance extension method.

as N_x becomes large. This is caused by the contribution of elements in the edge region to the radiation field becomes relatively small when N_x increases. These results mean that the edge effects on the actual gain pattern of the huge array can be ignored.

5. Conclusion

In this paper, properties of the periodic array antenna are reviewed. The impedance extension method based on these properties is also proposed as an approximate method for the huge array antenna. In the impedance extension method, the active impedance of the small array antenna obtained by MoM is extended to that of the huge array antenna. This method has two advantages. First, the CPU time and computer memory for analysis are independent of the array scale. Second, when the active impedance of the small array is once obtained, any huge array can be analyzed by the proposed method as long as it has the same array parameter excluding the number of elements N_x because the active impedance is almost independent of a number of elements and distribution of feeding voltage. To validate the impedance extension method, the active impedance and the actual gain pattern of the huge array are obtained by proposed method and high accuracy is confirmed. Furthermore, it is also found that edge effects on the actual gain pattern of the huge array can be ignored from the relative error estimation.

Since the model used in this paper is one-dimensional parallel dipole array antenna over a ground plane, it is required to investigate the validity of the proposed method for the case of a collinear dipole array antenna, where the distribution of the active impedance could be oscillatory even in the central region of the array antenna. Investigation of two-dimensional array antenna composed of a huge number of elements is also necessary to design the huge array antenna, in which accelerated MoM such as CG-FMM-FFT is required rather than the direct MoM as the full-wave analysis for comparison. Investigations of the collinear and two-dimensional array antennas are remaining problems to be solved in the future.

References

[1] A.K.M. Baki, K. Hashimoto, N. Shinohara, T. Mitani, and H. Matsumoto, "Isosceles-trapezoidal-distribution edge tapered array antenna with unequal element spacing for solar power satellite," IEICE Trans. Commun., vol.E91-B, no.2, pp.527–535, Feb. 2008.
[2] A. Ishimaru, R.J. Coe, G.E. Miller, and W.P. Geren, "Finite periodic structure approach to large scanning array problems," IEEE Trans. Antennas Propag., vol.AP-33, no.11, pp.1213–1220, Nov. 1985.
[3] R.C. Hansen and D. Gammon, "Standing waves in scan impedance of finite scanned arrays," Microw. Opt. Technol. Lett., vol.8, no.4, pp.175–179, March 1995.
[4] R.C. Hansen and D. Gammon, "Standing waves in scan impedance: E-plane finite array," Microw. Opt. Technol. Lett., vol.11, no.1, pp.26–32, Jan. 1996.
[5] R.C. Hansen and D. Gammon, "A gibbsian model for finite scanned arrays," IEEE Tans. Antennas Propag., vol.44, no.2, pp.243–248, Feb. 1996.
[6] R.C. Hansen, "Finite array scan impedance gibbsian models," Radio Science, vol.31, no.6, pp.1631–1637, Nov.-Dec. 1996.
[7] R.C. Hansen, Phased Array Antennas, Sect. 8.3, John Wiley & Sons, 1998.
[8] R.C. Hansen, "Anomalous edge effects in finite arrays," IEEE Trans. Antennas Propag., vol.47, no.3, pp.549–554, March 1999.
[9] R.F. Harrington, Field Computation by Moment Methods, Macmillan, New York, 1968.
[10] J.H. Richmond and N.H. Greay, "Mutual impedance of nonplanar-skew sinusoidal dipoles," IEEE Trans. Antennas Propag., vol.23, no.5, pp.412–414, May 1975.
[11] Q. Chen, Q. Yuan, and K. Sawaya, "Fast algorithm for solving matrix equation in MoM analysis of large-scale array antennas," IEICE Trans. Commun., vol.E85-B, no.11, pp.2482–2488, Nov. 2002.
[12] Q. Chen, Q. Yuan, and K. Sawaya, "Convergence of SOR in MoM analysis of array antenna," IEICE Trans. Commun., vol.E88-B, no.5, pp.2220–2223, May 2005.
[13] T.K. Sarker and S.M. Rao, "The application of the conjugate gradient method for the solution of electromagnetic scattering from arbitrarily oriented wire antennas," IEEE Trans. Antennas Propag., vol.AP-32, no.4, pp.398–403, April 1984.
[14] T.K. Sarker, "The conjugate gradient method as applied to electromagnetic field problems," IEEE Antennas Propagation Society Newsletter, vol.28, no.4, pp.4–14, Aug. 1986.
[15] R. Coifuman, V. Rokhlin, and S. Wandzura, "The fast multipole method for the wave equation: a pedestrian prescription," IEEE Antennas Propag. Mag., vol.35, no.3, pp.7–12, June 1993.
[16] V. Rokhlin, "Rapid solution of integral equations of scattering theory in two dimension," J. Comput. Phys., vol.86, no.2, pp.414–439, Feb. 1990.
[17] J.M. Song and W.C. Chew, "Multilevel fast-multipole algorithm for solving combined field integral equations of electromagnetic scattering," Microw. Opt. Technol. Lett., vol.10, no.1, pp.14–19, Sept. 1995.
[18] H. Zhai, Q. Yuan, Q. Chen, and K. Sawaya, "A numerical study on large-scale periodic array antenna by FMM and FFT," Proc. ISAP2006, pp.4035–4038, Singapore, July 2006.
[19] H. Zhai, Q. Chen, Q. Yuan, K. Sawaya, and C. Liang, "Analysis of large-scale periodic array antennas by CG-FFT combined with equivalent sub-array preconditioner," IEICE Trans. Commun., vol.E89-B, no.3, pp.922–928, March 2006.
[20] H. Zhai, Q. Yuan, Q. Chen, and K. Sawaya, "Preconditioners for CG-FMM-FFT implementation in EM analysis of large-scale periodic array antennas," IEICE Trans. Commun., vol.E90-B, no.3, pp.707–710, March 2007.

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