Optimization of Block Size for CBFM in MoM

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***Abstract*—The optimum number of blocks of the CBFM is derived theoretically and demonstrated numerically. The minimum order of CPU time for the CBFM is shown to be $O(N^{7/3})$, where N is the number of unknowns. In addition, the optimum size of overlapping region of the blocks also is investigated numerically.**

***Index Terms*—Characteristic basis function method, method of moments.**

I. Introduction

METHOD of moments (MoM) is well known as one of the powerful techniques for computational electromagnetics [1], [2]. In the MoM, unknown current on antennas or scatterers is obtained by solving $N \times N$ matrix equation,

$$\mathbf{V} = \mathbf{ZI} \tag{1}$$

where N is the number of unknown current coefficients, $\mathbf{V}$ is the N-dimensional voltage vector, $\mathbf{Z}$ is the $N \times N$ impedance matrix, and $\mathbf{I}$ is the N-dimensional unknown current vector. In general, the matrix (1) can be solved by multiplying $\mathbf{Z}^{-1}$ by both sides of (1). However, it is well known that CPU time required for calculating $\mathbf{Z}^{-1}$ is $O(N^3)$ by using direct matrix solving method, such as the Gauss-Jordan method, and $\mathbf{Z}^{-1}$ cannot be obtained for large-scale problems easily. Therefore, reduction of CPU time is indispensable for analysis of large-scale problems.

The iterative technique, such as conjugate gradient (CG) method, has been proposed as a promising technique for CPU time reduction in the MoM [3], [4]. Because CPU time per iteration of the CG method is $O(N^2)$, total CPU time is smaller than $O(N^3)$ when the number of iterations is smaller than N. However, the number of iterations in the CG method can be as large as $O(N)$ for an ill-conditioned problem [5]. Some large-scale problems are ill-conditioned (e.g., at low frequency [6]), and total CPU time of the CG method for analysis of such problems is still $O(N^3)$. Therefore, additional techniques must be introduced to reduce the CPU time of the CG method.

Multi-level fast multipole method (MLFMM) with efficient preconditioner has been introduced to the CG method for reduction of CPU time per iteration and number of iterations [7]–[9]. CPU time per iteration of the CG method becomes $O(N \log N)$ by introducing the MLFMM, and number of iterations can be reduced by the preconditioner. However, algorithm of the MLFMM is very complicated, and coding also is difficult. Moreover, effect of the preconditioner depends on the type of problems as well as several parameters, and it is difficult to realize constant number of iterations by introducing the preconditioner in general. Therefore, different approach is required for fast, stable, and simple solver.

Various fast direct solver, which can realize fixed and short execution time, also has been proposed. Coppersmith and Winograd have proposed $O(CN^{2.376})$ algorithm to solve matrix equation, where C is a constant coefficient [10]. However, constant coefficient C of the algorithm is large, and the algorithm is not efficient in practice. Compressed block decomposition (CBD) also has been proposed as a fast direct solver [11]. It has been reported that the CBD is an efficient algorithm for ill-conditioned problem at low frequency, and total CPU time is $O(N^2)$ [12]. Although the CBD is one of the fastest direct solver, considerably complicated algebraic operation is needed for the CBD.

As a fast, stable, and simple algorithm for the large-scale and ill-conditioned problem, characteristic basis function method (CBFM) also has been proposed [13]. In the CBFM, $\mathbf{Z}$ is compressed into $M^2 \times M^2$ smaller size of matrix (reduced matrix) by using characteristic basis function (CBF) and Galerkin's procedure, where M is the number of blocks. The reduced matrix can be easily inverted by LU decomposition or Gauss-Jordan method because $M^2 \ll N$. Because the CBFM does not include iterative procedure, the CBFM can reduce CPU time for analysis of the ill-conditioned problem. In addition, the CBFM is directly applicable to the analysis of a complicated model including dielectric object because algorithm of the CBFM is based on simple algebraic operation.

Many studies have been carried out for improvement of the CBFM. Singular value decomposition has been introduced to reduce lengthy CBF and improve accuracy of the CBFM [14], [15]. For reduction of CPU time, adaptive cross approximation (ACA) technique [16] and multilevel approach were utilized [17]. In recent years, the CBFM also has been applied to some complicated problems such as large-scale rough terrain profiles [18], helicopter with partial modifications of the rotor blades [19], and connected patch array on dielectric substrate [20]. In the CBFM, the number of blocks M and the size of the overlapping region, which improves accuracy of the CBFM, are known to be key parameters, which determine CPU time as well as accuracy. However, the optimum number of blocks and the size of the overlapping region, which gives the minimum CPU time

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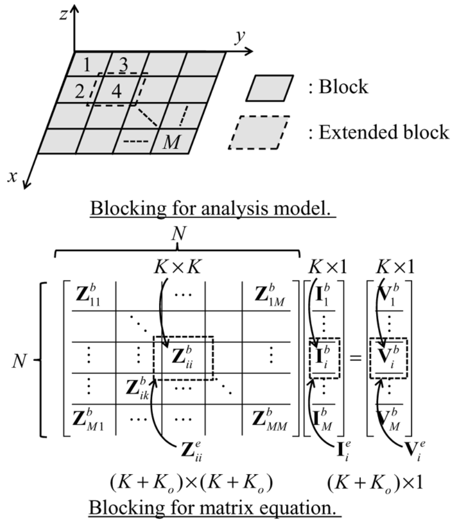


Fig. 1. Analysis of planar antenna by CBFM.

with enough accuracy, have not been clearly discussed in previous research.

In this paper, the optimum number of blocks in the CBFM is derived theoretically. The results of numerical simulation also is presented to show that the optimum number of blocks minimizes CPU time of the CBFM even when the overlapping region is introduced. Moreover, the size of the overlapping region also is optimized numerically from the view point of CPU time and accuracy. Finally, validity of the optimized CBFM is demonstrated by numerical simulation for three types of analysis models.

This paper is organized as follows. Section II reviews the principle of the CBFM. After that, computational cost of each procedure in the CBFM is tabulated, and the optimum number of blocks M is derived theoretically in Section III. In Section IV, numerical simulation is carried out to demonstrate the validity of the derived optimum number of blocks. Overlapping region between blocks also is optimized numerically. In addition, CPU time of optimized CBFM is compared with that of the CG method and Gauss-Jordan method. Section V concludes this paper.

II. Characteristic Basis Function Method

In this section, the principle of the CBFM proposed in [13] is briefly reviewed for the discussion of the optimum number of blocks. As an example, a planar antenna shown in Fig. 1 is selected as an analysis model to explain the principle of the CBFM, where N is the total number of unknown current coefficients, M is the number of blocks, K is the number of segments in a block, and K_0 is the number of overlapping segments in an extended block. $\mathbf{Z}_{ik}^b$ is $K \times K$ block self/mutual impedance matrix; $\mathbf{V}_i^b$ and $\mathbf{I}_i^b$ are K-dimensional block voltage and current vector, where superscript "b" means block, which does not include overlapping segments, whereas "e" means extended block, which includes overlapping segments.

In the CBFM, the analysis model is divided into M blocks as shown in Fig. 1. As a result, impedance matrix $\mathbf{Z}$, voltage vector $\mathbf{V}$, and current vector $\mathbf{I}$ are divided into $M \times M$, M, and M blocks, respectively. Unknown block current can be expressed by using CBF and weighting coefficient as follows:

$$\mathbf{I}_i^b = \sum_{k=1}^{M} \alpha_{(i,k)} \mathbf{J}_{(i,k)}^b \qquad i = 1, 2, \ldots, M \tag{2}$$

where $\mathbf{J}_{(i,k)}^b$ is the kth CBF in ith block and $\alpha_{(i,k)}$ is weighting coefficient for corresponding CBF. The CBF is called "primary basis" when $i = k$ and "secondary basis" when $i \neq k$. Expression (2) means the original problem for obtaining N-dimensional unknown current coefficients is transformed into the problem for obtaining M^2 CBFs and its weighting coefficients, where $M^2 \ll N (M \ll K)$ is required.

First, the primary basis of all M blocks is obtained in the CBFM. In general, the primary basis has the largest magnitude in all CBFs and strongly affects the accuracy of the final results. Therefore, the primary basis should be calculated carefully. The primary basis can be calculated accurately by the following $(K + K_0) \times (K + K_0)$ extended block matrix equation together with overlapping segments [13].

$$\mathbf{Z}_{ii}^e \mathbf{J}_{(i,i)}^e = \mathbf{V}_i^e \quad i = 1, 2, \ldots, M. \tag{3}$$

Because size of $\mathbf{Z}_{ii}^e$ is much smaller than that of the original $\mathbf{Z}$, (3) can be solved directly using the LU decomposition or Gauss-Jordan method. Calculated $(\mathbf{Z}_{ii}^e)^{-1}$ is stored in hard disk for the calculation of secondary basis. On the extended block current vector $\mathbf{J}_{(i,i)}^e$ obtained from (3), K_0 components corresponding to overlapping segments are discarded, and the remaining K components are stored as the primary basis. Overlapping segments are introduced to compensate current continuity between segments in adjacent blocks and can improve the primary basis because fictitious edges caused by block division are removed by overlapping segments. However, quite many overlapping segments cause more than enough CPU time of the CBFM. Therefore, size of the overlapping region should be optimized from the view point of both CPU time and the accuracy.

Next, secondary basis is obtained as follows:

$$\begin{aligned} \mathbf{Z}_{ii}^e \mathbf{J}_{(i,k)}^e = \mathbf{V}_{(i,k)}^e \text{ where } \mathbf{V}_{(i,k)}^e &= -\mathbf{Z}_{ik}^{e'} \mathbf{J}_{(k,k)}^{b'} \\ &\times (k = 1, 2, \ldots, i-1, i+1, \ldots, M), \end{aligned} \tag{4}$$

where $\mathbf{Z}_{ik}^{e'}$ is $(K + K_0) \times K'$ block matrix in $\mathbf{Z}_{ik}^e$; $\mathbf{J}_{(k,k)}^{b'}$ is K' components of the primary basis $\mathbf{J}_{(k,k)}^b$, where $K' = (K - K_0^{ik})$; and K_0^{ik} is the number of overlapping segments between ith block and kth block. As described above, $(\mathbf{Z}_{ii}^e)^{-1}$ stored previously is used to solve (4). As is the case with calculation of the primary basis, K_0 components of the extended block current vector $\mathbf{J}_{(i,k)}^e$ corresponding to overlapping segments are discarded, and the remaining K components are stored as the secondary basis. Totally, M^2 CBFs can be obtained using (3) and (4), but these CBFs are not always orthonormal basis. Therefore, Gram-Schmidt orthonormalization is applied to CBFs.

TABLE I
ORDER OF CPU TIME IN EACH PROCESS OF THE CBFM

Matrix filling time	$O(N^2)$
Calculation of Primary basis	$O(N^3/M^2)$
Calculation of Secondary basis	$O(N^2)$
Gram-Schmidt orthonormalization	$O(M^2N)$
Calculation of $\mathbf{u}_{(i,k)}$	$O(MN^2)$
Calculation of reduced matrix	$O(M^4N)$
Inversion of reduced matrix	$O(M^6)$

By using CBFs, the original matrix equation is transformed into the following equation.

$$\sum_{i=1}^{M}\sum_{k=1}^{M}\alpha_{(i,k)}\mathbf{u}_{(i,k)} = \mathbf{V}$$
$$\times(\mathbf{u}_{(i,k)} = [[\mathbf{Z}^b_{1i}\mathbf{J}^b_{(i,k)}][\mathbf{Z}^b_{2i}\mathbf{J}^b_{(i,k)}]\ldots[\mathbf{Z}^b_{Mi}\mathbf{J}^b_{(i,k)}]]^T). \quad (5)$$

Applying Galerkin's method, inner products of both sides of (5) with $\mathbf{u}^*_{(q,l)}$ are taken, and the original matrix equation is compressed into $M^2 \times M^2$ reduced matrix, where $M^2 \ll N$. Because the size of the $M^2 \times M^2$ reduced matrix is much smaller than that of the original matrix, the reduced matrix can be solved by LU decomposition or Gauss-Jordan method. Finally, unknown current vector is obtained, after weighting coefficient $\alpha_{(i,k)}$ calculated by the reduced matrix equation and CBFs are substituted into the expression (2).

III. BLOCK SIZE OPTIMIZATION

The CPU time of each process in the CBFM is shown as a function of M in Table I. The optimum number of blocks M_o, which minimizes the total CPU time can be derived. For the simplicity, the sum of the only two terms, which have highest/lowest order of M are considered for optimization. As shown in Table I, the CPU time for calculating primary basis has the lowest order of M, and that for inversion of reduced matrix has the highest order of M. However, CPU time for the calculating reduced matrix has practically the highest order of M than that for inversion of reduced matrix under the condition of $M^2 \ll N$ to reduce the size of the matrix by the CBFM. Based on the above consideration, the optimum number of blocks can be derived by

$$\frac{d}{dM}\left(\frac{N^3}{M^2} + M^4N\right) = 0 \quad (6)$$

which yields the optimum number M_o as

$$M_o \approx 0.9N^{1/3}. \quad (7)$$

As a result, the minimum CPU time is proportional to $N^3/M_o^2 + M_o^4N = N^{7/3}$. It should be noted that the effect of overlapping region is ignored in the above discussion and is investigated by the following numerical simulation. In addition, Nyquist's sampling theorem is clearly violated in (7), and it is inevitable to include some errors on the current obtained by the CBFM when $M = M_o$. Therefore, the block size M should be larger than M_o when the error on the current has to remain constant. However, results of numerical simulation in Section IV show that the error caused by the violation of the Nyquist's sampling theorem is small and tolerated value on scattering problems.

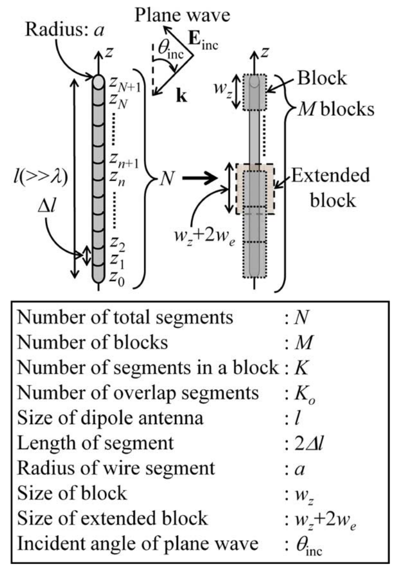


Fig. 2. Linear dipole antenna.

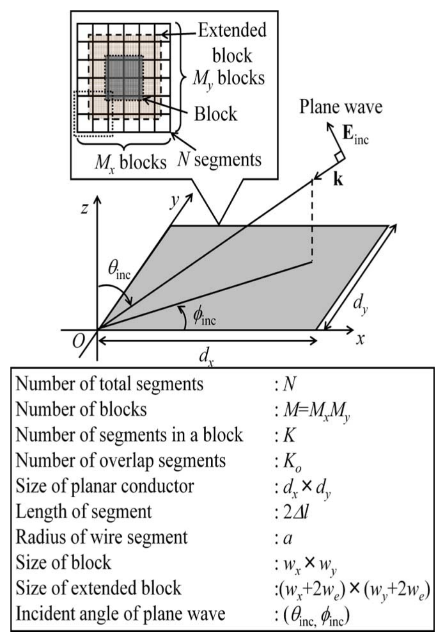
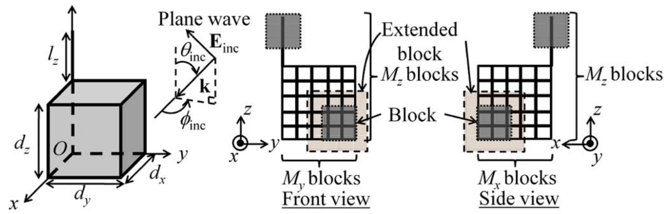


Fig. 3. Planar scatterer.

IV. NUMERICAL ANALYSIS

Numerical simulation is carried out for the three types of different scattering problems, that is, a linear dipole antenna, a planar scatterer, and a dipole antenna on a conducting box shown in Figs. 2–4. Richmond's MoM is used in this paper, and all conducting surfaces are divided into wire grid segments [2]. Size of the overlapping region is determined by extended width



Number of total segments	: N
Number of blocks	: $M=M_xM_yM_z$
Number of segments in a block	: K
Number of overlap segments	: K_o
Size of conducting box and dipole antenna	: $d_x \times d_y \times d_z$, l_z
Length and radius of wire segment	: $2\Delta l$, a
Size of block	: $w_x \times w_y \times w_z$
Size of extended block	: $(w_x+2w_e) \times (w_y+2w_e) \times (w_z+2w_e)$
Incident angle of plane wave	: $(\theta_{inc}, \phi_{inc})$

Fig. 4. Dipole antenna on conducting box.

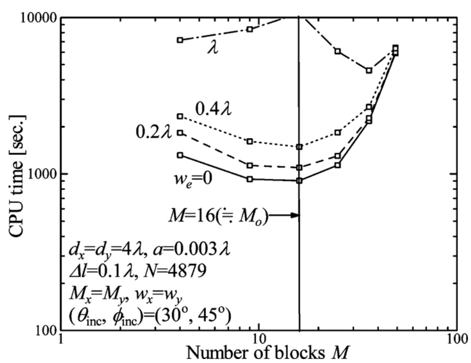


Fig. 5. Relation between number of blocks and CPU time on planar scatterer.

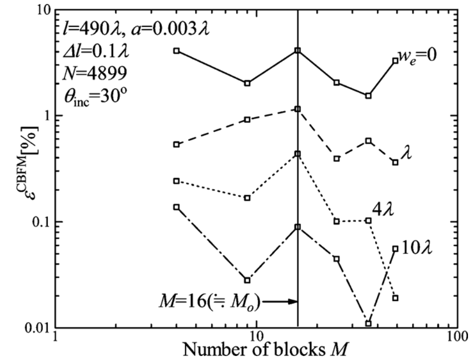


Fig. 6. Relation among extended width w_e, number of blocks M, and relative error for linear dipole antenna.

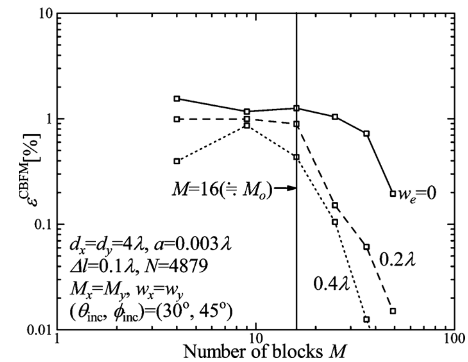


Fig. 7. Relation among extended width w_e, number of blocks M, and relative error for planar scatterer.

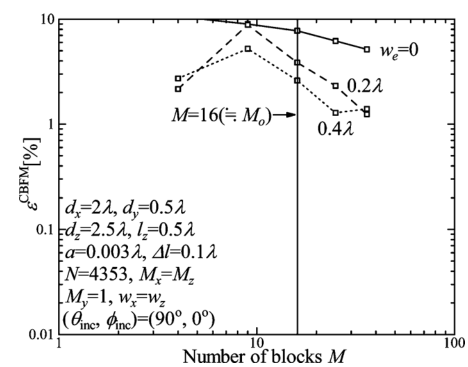


Fig. 8. Relation among extended width w_e, number of blocks M, and relative error for dipole antenna on conducting box.

w_e. A desktop computer with Pentium 4 3.6 GHz CPU and 2 GB RAM was used for all numerical calculation.

Relation between the number of blocks and CPU time for planar antenna is shown in Fig. 5 as a function of the number of the blocks M. It is found that $M = M_o$ gives the minimum CPU time when moderate overlapping region is employed. The same results are obtained for the analyses of the linear dipole antenna and dipole antenna on conducting box but are omitted here. Therefore, it can be said that $M = M_o$ is the optimum number of blocks, which minimizes CPU time of the CBFM.

In the above discussion, the number of blocks M is optimized from the view point of CPU time. However, accuracy also should be considered for the optimization of the number of blocks. Moreover, the size of the overlapping region, which improves accuracy of the final results, should be optimized because a large overlapping region causes increase of CPU time as shown in Fig. 5 ($w_e = \lambda$). The number of blocks M and size of overlapping region are optimized from the view point of the accuracy.

There are two key parameters, which determine the accuracy of results obtained by the CBFM; extended width w_e and number of blocks M. Extended width determines the size of overlapping region and improves the accuracy of the results. The number of blocks M determines the size of reduced matrix, and the current obtained from the reduced matrix can include some errors by violation of the Nyquist's sampling theorem. The accuracy of the results obtained by the CBFM is evaluated by the following relative error.

$$\varepsilon^{CBFM} = \frac{1}{N}\sqrt{\sum_{i=1}^{N} \frac{\left|I_i^{GJ} - I_i^{CBFM}\right|^2}{\left|I_i^{GJ}\right|^2}} \tag{8}$$

where I_i^{GJ} and I_i^{CBFM} are the current of ith segment obtained by the Gauss-Jordan method and CBFM, respectively.

Relation among extended width w_e, number of blocks M, and the relative error is shown in Figs. 6–8. In all cases, the relative error of the CBFM decreases because of the presence of the overlapping region. A few percent relative error of the current can be tolerated in many cases, and the acceptable relative error can be realized by modification of extended width w_e even

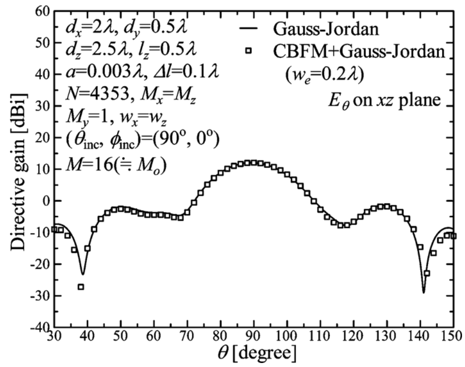


Fig. 9. Scattering pattern of dipole antenna on conducting box.

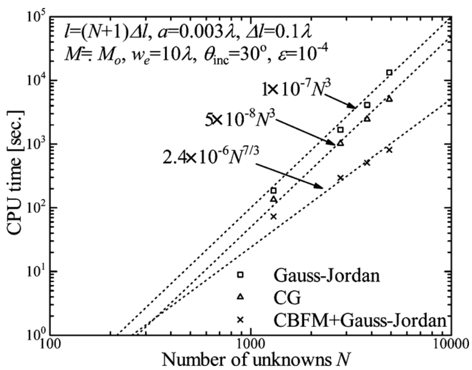


Fig. 10. CPU time for analysis of linear dipole antenna.

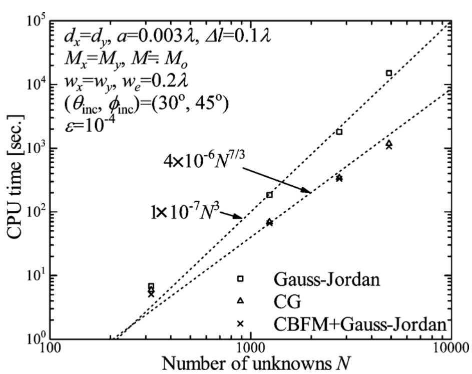


Fig. 11. CPU time for analysis of planar scatterer.

when $M = M_o$ as can be seen in Figs. 7 and 8. Therefore, the optimum number of blocks also is determined as $M = M_o$ from the view point of accuracy.

On the extended width, the number of overlapping segments in linear dipole antenna is small even when $w_e = 10\lambda$ because of one-dimensional segment geometry. Therefore, for linear dipole antenna, $w_e = 10\lambda$ can be considered as the optimum extended width, which can realize not only accurate results but also small CPU time. On the other two structures, $w_e = 0.2\lambda$ is selected as the optimum extended width in this paper.

Scattering pattern of dipole antenna on conducting box obtained using the optimized CBFM is shown in Fig. 9. As compared with the scattering pattern obtained by the Gauss-Jordan method, small error occurs around a null, but almost the same pattern is obtained by the optimized CBFM. The CPU time for

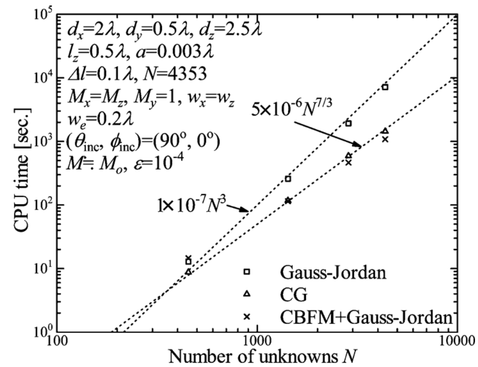


Fig. 12. CPU time for analysis of dipole antenna on conducting box.

the analysis of each structure is shown in Figs. 10–12. Convergence criteria of the CG method is relative residual $\epsilon = 10^{-4}$. The CPU time for the analysis of the linear dipole antenna by the CG method is $O(N^3)$ because the problem is known as an ill-conditioned problem [5]. On the other hand, the CPU time required for analysis by the optimized CBFM is $O(N^{7/3})$, which means that the optimized CBFM is more effective than the CG method for the analysis of the ill-conditioned problem. The remaining two problems are not ill-conditioned problems, and CG method is effective for the reduction of the CPU time. However, it is found that the optimized CBFM is as effective as the CG method for the reduction of CPU time.

V. CONCLUSION

In this paper, number of blocks and size of the overlapping region between blocks in the CBFM were optimized from the view point of accuracy and CPU time. The optimum number of blocks, which gives the minimum CPU time of the CBFM was derived theoretically as $M \approx 0.9N^{1/3}$, and minimum CPU time is $O(N^{7/3})$. Numerical analysis on three types of different structures was carried out to show that the optimum number of blocks derived theoretically can realize minimum CPU time. In addition, the size of the overlapping region between blocks was optimized by the numerical simulation. Finally, it was pointed out that the optimum CBFM can realize minimum CPU time without large loss of accuracy.

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REFERENCES

[1] R. F. Harrington, *Field Computation by Moment Methods*. New York: Macmillan, 1968.
[2] J. H. Richmond and N. H. Geary, “Mutual impedance of nonplanar-skew sinusoidal dipoles,” *IEEE Trans. Antennas Propag.*, vol. 23, no. 5, pp. 412–414, May 1975.
[3] T. K. Sarkar and S. M. Rao, “The application of the conjugate gradient method for the solution of electromagnetic scattering from arbitrarily oriented wire antennas,” *IEEE Trans. Antennas Propag.*, vol. 32, no. 4, pp. 398–403, Apr. 1984.
[4] A. F. Peterson and R. Mittra, “Convergence of the conjugate gradient method when applied to matrix equations representing electromagnetic scattering problems,” *IEEE Trans. Antennas Propag.*, vol. AP-34, no. 12, pp. 1447–1454, Dec. 1986.

[5] K. Konno, Q. Chen, and K. Sawaya, “Quantitative evaluation for computational cost of CG-FMM on typical wiregrid models,” *IEICE Trans. Commun.*, vol. E93-B, no. 10, pp. 2611–2618, Oct. 2010.

[6] J. S. Zhao and W. C. Chew, “Integral equation solution of Maxwell’s equations from zero frequency to microwave frequencies,” *IEEE Trans. Antennas Propag.*, vol. 48, no. 10, pp. 1635–1645, Oct. 2000.

[7] J. M. Song and W. C. Chew, “Multilevel fast-multipole algorithm for solving combined field integral equations of electromagnetic scattering,” *Microw. Opt. Technol. Lett.*, vol. 10, no. 1, pp. 14–19, Sep. 1995.

[8] J. M. Song, C. C. Lu, and W. C. Chew, “Multilevel fast multipole algorithm for electromagnetic scattering by large complex objects,” *IEEE Trans. Antennas Propag.*, vol. 45, no. 10, pp. 1488–1493, Oct. 1997.

[9] J. Lee, J. Zhang, and C. C. Lu, “Sparse inverse preconditioning of multilevel fast multipole algorithm for hybrid integral equations in electromagnetics,” *IEEE Trans. Antennas Propag.*, vol. 52, no. 9, pp. 2277–2287, Sep. 2004.

[10] D. Coppersmith and S. Winograd, “Matrix multiplication via arithmetic progressions,” *J. Symbolic Comput.*, vol. 9, no. 3, pp. 251–280, Mar. 1990.

[11] A. Heldring, J. M. Rius, J. M. Tamayo, J. Parrón, and E. Úbeda, “Fast direct solution of method of moments linear system,” *IEEE Trans. Antennas Propag.*, vol. 55, no. 11, pp. 3220–3228, Nov. 2007.

[12] A. Heldring, J. M. Rius, J. M. Tamayo, J. Parrón, and E. Úbeda, “Multiscale compressed block decomposition for fast direct solution of method of moments linear system,” *IEEE Trans. Antennas Propag.*, vol. 59, no. 2, pp. 526–536, Feb. 2011.

[13] V. V. S. Prakash and R. Mittra, “Characteristic basis function method: A new technique for efficient solution of method of moments matrix equations,” *Microw. Opt. Technol. Lett.*, vol. 36, no. 2, pp. 95–100, Jan. 2003.

[14] G. Tiberi, M. Degiorgi, A. Monorchio, G. Manara, and R. Mittra, “A class of physical optics-SVD derived basis functions for solving electromagnetic scattering problems,” in *Proc. IEEE AP-S Int. Symp.*, Jul. 2005, pp. 143–146.

[15] M. Degiorgi, G. Tiberi, A. Monorchio, G. Manara, and R. Mittra, “An SVD-based method for analyzing electromagnetic scattering from plates and faceted bodies using physical optics bases,” in *Proc. IEEE AP-S Int. Symp.*, Jul. 2005, pp. 147–150.

[16] R. Maaskant, R. Mittra, and A. Tijhuis, “Fast analysis of large antenna arrays using the characteristic basis function method and the adaptive cross approximation algorithm,” *IEEE Trans. Antennas Propag.*, vol. 56, no. 11, pp. 3440–3451, Nov. 2008.

[17] J. Laviada, F. Las-Heras, M. R. Pino, and R. Mittra, “Solution of electrically large problems with multilevel characteristic basis functions,” *IEEE Trans. Antennas Propag.*, vol. 57, no. 10, pp. 3189–3198, Oct. 2009.

[18] A. Yagbasan, C. A. Tunc, V. B. Ertürk, A. Altintas, and R. Mittra, “Characteristic basis function method for solving electromagnetic scattering problems over rough terrain profiles,” *IEEE Trans. Antennas Propag.*, vol. 58, no. 5, pp. 1579–1589, May 2010.

[19] J. Laviada, J. G. Meana, M. R. Pino, and F. Las-Heras, “Analysis of partial modifications on electrically large bodies via characteristic basis functions,” *IEEE Antennas Wireless Propag. Lett.*, vol. 9, pp. 834–837, 2010.

[20] S. G. Hay, J. D. O’Sullivan, and R. Mittra, “Connected patch array analysis using the characteristic basis function method,” *IEEE Trans. Antennas Propag.*, vol. 59, no. 6, pp. 1828–1837, Jun. 2011.

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